Packing trees of bounded diameter into the complete graph

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Abstract

Let d be a positive integer. We prove that there exists a constant $c = c(d)$ such that if $T_1, \ldots, T_n$ is a sequence of trees such that $|V(T_i)| = i$, $\mathrm{diam}(T_i) \leq d + 2$, and there exists $x_i \in V(T_i)$ such that $T_i - x_i$ has at least $(1 - c)(i - 1)$ isolated vertices, then $T_1, \ldots, T_n$ can be packed into K_n. This verifies a special case of the Tree Packing Conjecture. We then prove that if T is a tree of order $n + 1$ and there exists $x \in V(T)$ such that $T - x$ has at least $n - \sqrt{n}/8$ isolated vertices, then $2n + 1$ copies of T may be packed into K_{2n+1}. Finally, we show that there exists a constant $c' = c'(d)$ such that if T is a tree of order $n + 1$, $\mathrm{diam}(T) \leq d + 2$, and there exists $x \in V(T)$ such that $T - x$ has at least $(1 - c')n$ isolated vertices, then $2n + 1$ copies of T may be packed into K_{2n+1}. The last two results verify special cases of Ringels conjecture.