

# Graph homomorphism into an odd cycle

Hong-Jian Lai      Gexin Yu \*

Department of Mathematics, West Virginia University  
Morgantown, WV, 26505, USA

E-mail: hjlai@math.wvu.edu, yu@math.wvu.edu

## Abstract

For graphs  $G$  and  $H$ , a map  $f : V(G) \mapsto V(H)$  is a homomorphism if  $f$  preserves adjacency. Let  $Hom(G, H)$  denote the set of all homomorphisms from  $G$  into  $H$ . In this paper, we proved that for a simple graph  $G$  with  $n = |V(G)|$  and for  $k$  with  $n \geq k \geq 5$ , if the odd girth of  $G$  is at least  $2k + 1$  and if the minimum degree  $\delta(G) > 2n/(2k + 3)$ , then  $Hom(G, Z_{2k+1}) \neq \emptyset$ , where  $Z_{2k+1}$  denotes the cycle of length  $2k + 1$ . As a corollary, we settled affirmatively the following open problem posted by Albersson, Chan and Haas in 1993: If a graph  $G$  satisfies the conditions above, must the independence of  $G$ , which is the ratio of the independence number of  $G$  to the number of vertices of  $G$ , be at least  $k/(2k + 1)$ ?

---

\*Part of this paper is done in this author's Master thesis, and this author would like to give his special thanks to his Master superadvisor Professor Bolian Liu.