On non-z(mod k) dominating sets

Yair Caro
Department of Mathematics
University of Haifa - Oranim
Tivon - 36006, Israel
ya_caro@kvgeva.org.il

Michael S. Jacobson
Department of Mathematics
University of Louisville
Louisville, KY - 40292, USA
mikej@louisville.edu

Abstract

For a graph G, a positive integer $k, k \geq 2$, and a non-negative integer with $z < k$ and $z \neq 1$, a subset D of the vertex set $V(G)$ is said to be a *non-z(mod k) dominating set* if D is a dominating set and for all $x \in V(G), |N[x] \cap D| \not\equiv z(\text{mod } k)$.

For the case $k = 2$ and $z = 0$, it has been shown that these sets exist for all graphs. The problem for $k \geq 3$ is unknown (the existence for even values of k and $z = 0$ follows from the $k = 2$ case.) It is the purpose of this paper to show that for $k \geq 3$ and with $z < k$ and $z \neq 1$, that a non-z(mod k) dominating set exists for all trees. Also, it will be shown that for $k \geq 4, z \neq 1, 2$ or 3 that any unicyclic graph contains a non-z(mod k) dominating set. We also give a few special cases of other families of graphs for which these dominating sets must exist.