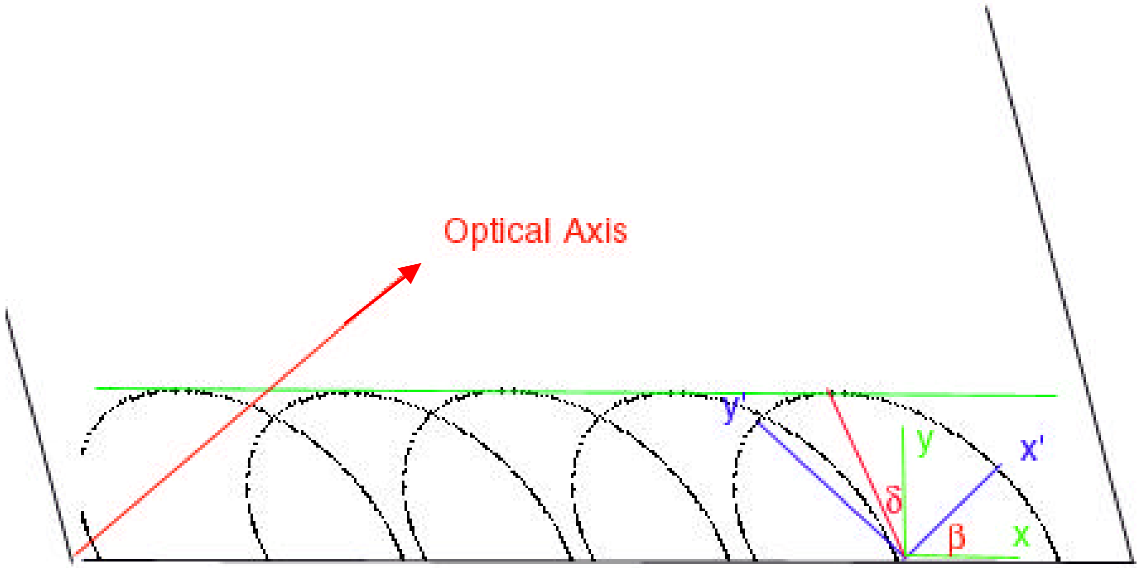
Prediction of the Deviation Angle for Calcite



Consider the extraordinary ray in the Huygen construction. These Huygen wavelets have polarization at every point which lie in the plane including the optical axis. The wavelets spread out at different rates depending on the direction of propagation. During a time Δt a wavelet expands a distance $\frac{c}{n_e}\Delta t$ in the y' direction, which is perpendicular to the optical axis, and $\frac{c}{n_0}\Delta t$ in the x' direction, which is parallel to the optical axis.

Thus the elliptical wavelet is of the form $K = \frac{y'^2}{\left(\frac{1}{n_e}\right)^2} + \frac{x'^2}{\left(\frac{1}{n_0}\right)^2} = c^2\Delta t^2$

These coordinates are related to the x,y coordinated system , where x is parallel to and y normal to the surface, via the relations:

$x = x' \cos\beta - y' \sin\beta$

$y = y' \cos + x' \sin\beta$

So:

$$K = \frac{1}{n_0^2}\left[x^2\cos^2\beta + 2xy\cos\beta\sin\beta + y^2\sin^2\beta \right]$$

$$+ \frac{1}{ne^2}\left[y^2\cos^2\beta - 2xy\cos\beta\sin\beta + x^2\sin^2\beta \right] = c^2\Delta t^2$$

The wavelet expanding from each point on the former wavefront determines the envelope of the new wavefront at just one point, namely the point on the ellipse which satisfies

$$\left(\frac{\partial x}{\partial y}\right)_{K=c^2\Delta t^2} = \frac{\left(\frac{-\partial K}{\partial y}\right)_x}{\left(\frac{\partial K}{\partial x}\right)_y} = \infty$$

i.e. when $\left.{\partial K}/{\partial x}\right)_y = 0$

so we require

$$\left(\frac{1}{n_0^2} - \frac{1}{n_e^2}\right) 2y\cos\beta\sin\beta + \left(\frac{\sin^2\beta}{n_e^2} + \frac{\cos^2\beta}{n_0^2}\right) 2x = 0$$

$$\frac{x}{y} = -\tan\delta = \frac{\left(\frac{1}{n_e^2} - \frac{1}{n_0^2}\right)\cos\beta\sin\beta}{\frac{\sin^2\beta}{n_e^2} + \frac{\cos^2\beta}{n_0^2}}$$

$$-\tan\delta = \frac{\left(\frac{1}{n_e^2} - \frac{1}{n_0^2}\right)}{\frac{\tan\beta}{n_0^2} + \frac{\cot\beta}{n_e^2}}$$

Find the value of δ using the values:

$\beta = 54.5°$

$n_e = 1.6548$

$n_o = 1.4864$