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LEARNING-THE-POTENTIAL BY DOING AND ASSET PRICE VOLATILITY*

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ABSTRACT

This paper provides a possible alternative explanation for the surge in high-tech stocks in the 1990s and their post-2000 crash. We model the experimentation process of a manager trying to learn the potential of a new technology. Given reasonable assumptions, this model yields a boom-bust cycle in stock prices. We derive measures of price volatility and the timing of the crash. Our comparative dynamics suggest that lower interest rates (caused, say, by expansionary monetary policy), accelerate the boom-bust cycle.

JEL classification: D83, G12

Keywords: High-tech stocks; bubbles; experimentation; learning-by-doing

1. INTRODUCTION

Research on asset price volatility has accelerated since the surge in US high-tech stock prices in the 1990s and their crash in 2000 (Figure 1). This boom in high-tech stocks has been characterized by many researchers as a bubble.[1] Though bubble theories are appealing, they depart from the standard economic assumption of efficient markets, where asset prices reflect fundamental values, as determined by the present value of expected future income streams. It may therefore be interesting to seek possible non-bubble explanations.

This paper builds a model of a boom-bust cycle in stock prices based on learning. For a new technology, its potential is not known immediately. For example, nobody was sure at first how many business opportunities the internet would create. To discover this potential, people had to experiment with the technology. We therefore start by modeling the learning process of a manager.[2] The firm has a Leontief production function, one of whose arguments is the unknown potential of the technology. The manager chooses the amount of a variable input, then, by observing the output, she learns something about the potential of the technology. If the output is less than the variable input, this reveals the actual full potential of the technology. If the output equals the variable input, this means that the technology's potential has not yet been exhausted, so the manager raises her expectation about the potential and inputs more next period. Before she learns the potential, the manager keeps increasing her expectation of it. There is boom in firm value. Once she learns the actual potential, firm value generally falls. That is, there is generally a crash when the technology's full potential is finally learned. Thus, even if asset prices follow fundamentals, there is a boom-bust cycle in stock prices.

A recent paper by Sampson (2003) also provides a learning-theoretic model of a boom-bust cycle. Roughly speaking, Sampson traces the median of an increasingly skewed distribution. He argues that the cycle relies entirely on a “Variance effect.” Other fundamentals-based discussions of the recent boom-bust cycle include Cochrane (2002), Edison and Slok (1999) and Bordo and Jeanne (2002).

Section 2 presents the basic learning model. Section 3 applies the model to the stock market and derives the boom-bust cycle. Section 4 further analyzes the properties of the model. Section 5 concludes.

2. THE BASIC MODEL

Assume the firm has a Leontief production function with two inputs, one variable factor, and one fixed factor. We refer to the fixed factor as the “potential” of the technology. The production relation is

$$q = \min(A, x),$$

where A is the true but unknown potential of the technology and x is the quantity of the variable factor. The manager chooses the amount of x to maximize firm value. The level of output also tells her something about the technology's potential, A. If $q < x$, she learns that $A = q$, while if $q = x$, she learns only that $A \geq x$, and so, becomes more optimistic.

Assume the potential of the technology has the Pareto distribution

$$f(a) = (b-1)L^{b-1}a^{-b} \qquad a \geq L, b > 2.$$

Assume $b > 2$, so the expected value of A bounded. Normalize the output price to one, let w be the unit cost of the variable factor, and ignore fixed costs for simplicity. This yields a per period profit function of

$$\min(A, x) - wx.$$

Assume $w < 1$ (variable cost less than price) so that it is not optimal to shut down. Then the one-period (myopic) problem would be to choose x to maximize

$$E\pi(x, L) = E\big[\min(A, x)\big] - wx \tag{1}$$

$$= \tfrac{b-1}{b-2} L - \tfrac{1}{b-2} L^{b-1} x^{2-b} - wx.$$

Maximizing this gives the myopic optimal solution

$$x_M^* = w^{-\frac{1}{b-1}} L, \tag{2}$$

which is greater than L because $w < 1$. Thus, if the manager's information about A is $A \geq L$, then she chooses an amount $x_M^* > L$ of the variable factor. Thus, in the next stage, the manager either knows $A \geq x_M^*$ if there was no shortage of the fixed factor, or the actual value of A if $A < x_M^*$. Thus there is always learning going on, even in myopic case.

Now consider a manager who is not myopic. We use Bellman's equation to model the manager's infinite horizon decision problem. Thus, suppose that at the very beginning, the manager does not know the value of A, but only knows $A \geq L$. Let the firm's value function be $V(L)$. The firm's fundamental value is the expected profit today plus the discounted expected present value of profits as of tomorrow. If $A \geq x$, then this expected present value will just be the value function evaluated at x, or $V(x)$. On the other hand, if $L \leq A < x$, then the manager will learn A, so she will simply hire $x = A$ units of the variable factor from next period on. Thus, the profit per period will be $(1 - w)A$, which, if discounted to today yields

$$\frac{\beta(1-w)A}{(1-\beta)}. \tag{3}$$

Putting this all together and maximizing over the firm's choice variable x, gives the Bellman's equation

(4)

$$V(L) = \max_x \Big\{ E\pi(x,L) + \beta \Big[P(A \geq x)V(x) + P(L \leq A < x) E\big[\frac{(1-w)A}{1-\beta} | L \leq A < x\big]\Big]\Big\}$$

$$= \max_x \Big\{ E\pi(x,L) + \beta \Big[P(A \geq x)V(x) + \tfrac{1-w}{1-\beta} \int_L^x af(a)da\Big]\Big\},$$

where $E\pi(x,L)$ is given in equation (1). Using equation (1), together with the functional form of the Pareto distribution, gives

(5)
$$V(L) = \max_x \Big\{ \frac{b-1}{b-2}L - \frac{1}{b-2}L^{b-1}x^{2-b} - wx$$
$$+\beta L^{b-1}x^{1-b}V(x) + \tfrac{\beta}{1-\beta}\tfrac{b-1}{b-2}(1-w)(L - L^{b-1}x^{2-b})\Big\}.$$

Now the form of (5) suggests that the right hand side is linear homogeneous in L and x if $V(x)$ is linear in x. Thus, let $V(x) = \Theta x$ and substitute this into (5). Divide both sides by L and let $y = x/L$, giving

(6)
$$\Theta = \max_y \Big\{ -wy - \Big[\frac{1}{b-2} + \frac{\beta}{1-\beta}\frac{b-1}{b-2}(1-w) - \beta\Theta\Big] y^{2-b} + \frac{b-1}{b-2}\frac{1-\beta w}{1-\beta}\Big\}.$$

The first order condition for (6) yields

(7)
$$y^* = \Big[\frac{1 + \frac{\beta}{1-\beta}(b-1)(1-w) - \beta(b-2)\Theta}{w}\Big]^{\frac{1}{b-1}}.$$

Equations (6) and (7) give two equations in two unknowns, $y = y^*$ and $\Theta = \Theta^*$. This, in turn, yields both the value function and the optimal choice of $x^* = y^*L$. While we cannot solve to get explicit formulas for y^* and Θ^*, it is straightforward to estimate them numerically. Figure 2 shows the graphs of equations (6) and (7).

3. APPLICATION TO STOCK PRICE VOLATILITY

In this section, we apply the basic learning model to the high-tech stock market. Assume that there is no principal-agent problem, so the manager simply maximizes the value of the firm. By assuming that investors have the same objective function, patience level, and information as the manager, both of them value firm the same. The market price is therefore the same as the firm value, V, maximized by the manager. Thus, in the following description, investors and the manager are interchangeable.

The potential of the new technology is unknown. Experimentation, if it does not exhaust that potential, provides good news, since it suggests that this potential is even higher than previously thought. Investors therefore raise their expectation of the potential of the technology. As long as the full potential has not been realized, investors allow for the possibility that this potential may be enormous. After the agents finally learn the potential, however, there is no more uncertainty and firm value usually drops. This yields a boom-bust cycle in stock prices.

3.1 The boom-bust cycle

Given the model setup in Section 2, the sequence of events is as follows: At the very beginning, the manager only knows that the potential of the technology is randomly drawn from a Pareto distribution with lower bound L. The firm value is then $V_1 = V(L)$. In period one, the manager chooses $x_1 = y^*L$ to maximize the firm value. At the end of period 1, the manager either observes output $q < x_1$, and so, learns that the actual value of A is $A = q < x_1$, or observes $q = x_1$, and so, learns that $A \geq x_1$. In the latter case, she increases the lower bound of the potential to

x_1. Thus at the beginning of period 2, the firm's value is $V_2 = V(x_1) > V_1$. Then the manager chooses $x_2 = y^* x_1$ to maximize firm value, and the process continues. Finally suppose that the manager learns the actual A during period t after choosing $x_t = y^* x_{t-1}$. Thus, $x_{t-1} \leq A < y^* x_{t-1}$. Then firm value changes from $V_t = \Theta x_{t-1}$ at the befinning of period t to $V_{t+1} = (1-w)A/(1-\beta)$ at the befinning of period $t+1$ where $x_{t-1} \leq A < y^* x_{t-1}$. This will generally lead to a crash.

During this experimentation process, before she learns A, the manager keeps increasing her expectation of the potential of the technology, $E(A|A \geq x_t) = [(b-1)/(b-2)]x_t$, as she increases the lower bound and carries out more aggressive experimentation. If this does not exhaust the potential, the firm value $V(x) = \Theta x$ keeps increasing as x does. We assume that stock price follows the fundamental value of the firm. This means that the stock price keeps increasing until investors actually learn A. Their rosy expectations then give way to reality. Firm value then often decreases, and the stock price crashes. That bust follows boom seems to be a common phenomenon in reality. In the present model, investors' rational expectations generate a boom-bust cycle of stock prices, even in an efficient market.

Figure 3 illustrates a typical path of firm values generated by our model. It shows a constant rise in firm value, followed by a sudden drop once the actually potential is learned. This boom-bust cycle, in essence, is similar to what we have observed in reality (Figure 1), except that, because of the setup of our model, firm value finally levels off after the drop.

3.2 The size of the crash

Define a new variable δ as change in firm value (stock price) in the period when the manager learns the true potential of the technology:

$$(8) \qquad \delta = V_{t+1} - V_t = \frac{(1-w)A}{1-\beta} - \Theta x_{t-1}, \quad \text{for} \quad A \in [x_{t-1}, x_t).$$

Here δ lies in the interval $\left[\left(\frac{1-w}{1-\beta} - \Theta\right)x_{t-1}, \left(\frac{1-w}{1-\beta}y^* - \Theta\right)x_{t-1}\right]$.

If δ is negative, learning A leads to a crash in stock prices. We derive the expectation of δ, and show that δ is usually negative, so stock prices usually crash when the potential, A, is learned.

First, the PDF of A, given that the manager actually learns A in period t, is

$$f(a|x_{t-1} \leq A < y^* x_{t-1}) = \frac{f(a)}{\int_{x_{t-1}}^{y^* x_{t-1}} f(u)du}$$
$$= \frac{(b-1)L^{b-1}a^{-b}}{L^{b-1}x_{t-1}^{1-b} - L^{b-1}(y^* x_{t-1})^{1-b}} = \frac{(b-1)a^{-b}}{x_{t-1}^{1-b}(1-y^{*(1-b)})}.$$

The expected value of A given that A is learned in period t, is then

$$E(A|A\ learned) = \int_{x_{t-1}}^{y^* x_{t-1}} af(a|x_{t-1} \leq A < y^* x_{t-1})da$$
$$= \frac{b-1}{b-2}\frac{1-y^{*(2-b)}}{1-y^{*(1-b)}}x_{t-1}.$$

The expected size of the price change, in the period that the manager learns A is then

$$(9) \qquad E(\delta|A\ learned) = \frac{1-w}{1-\beta}E(A|A\ learned) - \Theta x_{t-1}$$
$$= \left(\frac{1-w}{1-\beta}\frac{b-1}{b-2}\frac{1-y^{*(2-b)}}{1-y^{*(1-b)}} - \Theta\right)x_{t-1}.$$

The lower bound of δ is always negative, and the upper bound is often negative. If the upper bound *is* negative, then, when the manager learns the potential, the firm value always drops, hence the stock price crashes. Since every technology has bounded potential, the learning process then makes the boom-bust cycle inevitable. Revisiting equation (4) we can decompose the firm's value into two parts, the current profits and the present value of the expected future profits. The later part is a convex combination of two terms, that is, the firm value if the manager does not learn A and the firm value if the manager learns A. Of these later two terms, the first is larger. The expected firm value therefore is greater than the firm value when the manager learns A. On average there is a crash when A is learned.

However, for some parameter values, δ has a positive upper bound, which means that in some cases, firm value may level off or even increase in the period that A is learned. Thus learning the potential does not *necessary* cause a crash, though it usually does.

PROPOSITION 1: The expectation of stock price change, in the period when A is learned, is negative.

PROOF: NOT DONE YET.

3.3 Expected number of periods until A is learned

Compared to Sampson (2003) in which the time of the crash is a given parameter t, in our model the timing of the crash is a random variable. Define N as the number of periods until A is learned. The expectation of N can be calculated as follows:

$$
\begin{aligned}
E(N) &= 1\ P(L \le A < x_1) + 2\ P(x_1 \le A < x_2) + \cdots \\
&= \left[1\ \left(L^{1-b} - x_1^{1-b}\right) + 2\ \left(x_1^{1-b} - x_2^{1-b}\right) + \cdots\right]\ L^{b-1} \\
&= 1 + L^{b-1}\left(x_1^{1-b} + x_2^{1-b} + \cdots\right).
\end{aligned}
\tag{10}
$$

Now, before the crash, $x_t = y^* x_{t-1} = (y^*)^2 x_{t-2} = \cdots = (y^*)^t x_0 = (y*)^t L$. Using this in equation (10) gives

$$E(N) = 1 + L^{b-1}L^{1-b}\left[(y^*)^{(1-b)} + (y^*)^{2(1-b)} + \cdots\right] \tag{11}$$

$$= 1 + \tfrac{(y^*)^{1-b}}{1-(y^*)^{1-b}} = 1 + \tfrac{1}{(y^*)^{b-1}-1}.$$

The next section uses these formulas to examine the properties of this boom-bust cycle. Before going to these theoretical results, however, please refer to the nummerical examples in Table 1 for a brief view of how the model behaves.

For all the parameter values in Table 1, the expected percentage change in firm value, when A is learned, is negative, as in Proposition 1. Thus, on average, firm value drops and stock prices crash when the manager learns the actual potential of the technology. However, for some parameter values, the maximum possible δ is positive, so a positive price change is possible. This means that a boom is not *always* followed by a bust.

A higher discount factor, β, means that the manager is more patient. She experiments more, so she learns the technology's potential sooner. The expected number of periods until A is learned is therefore smaller. In addition, the price change is more likely to be negative, so a crash becomes more likely. It is also larger on average.

A higher wage rate, w, hinders the manager's experimentation, so the expected number of periods until A is learned is bigger. Also, the firm's value is more likely to fall when A is learned, and the crash gets bigger on average.

If the Pareto distribution parameter, b, gets bigger, the expected number of periods until A is learned gets bigger. Also, the percentage change of firm value when A is learned becomes less negative.

4. PROPERTIES OF THE MODEL

We examine the properties of the model by considering how the parameter values influence the boom-bust cycle.

4.1 The influence of the discount factor β

The discount factor β measures the patience level of the manager. The larger β, the more patient the manager and the more future profits weigh in firm value. If β is larger, the manager will experiment more aggressively, which enables her to learn the true potential, A, sooner.

PROPOSITION 2: The more patient the manager, the larger y^*, and so, the more the manager experiments.

PROOF: Note: We do not have the proof yet. Instead, below we prove that there is more experimentation when $\beta > 0$ than when $\beta = 0$.

When $\beta = 0$, equation (7) gives

$$y^* = w^{-\frac{1}{b-1}}, \quad \textit{and} \quad x^* = y^* L = w^{-\frac{1}{b-1}} L.$$

This agrees with the myopic result in Section 2.

Next we want to show that when the manager is patient, so $\beta > 0$, then

$$y^* > w^{-\frac{1}{b-1}}, \tag{12}$$

so the optimal choice, x^*, of the variable factor is larger in the case with patience. This will imply that patience speeds learning.

First, the firm's discounted future profits, if the manager knows A before the first production decision, would be

$$E\Big(\frac{(1-w)A}{1-\beta}\Big) = \frac{1-w}{1-\beta}E(A) = \frac{1-w}{1-\beta}\frac{b-1}{b-2}L.$$

This must be larger than the expected discounted profits resulting from the costly process of learning A through trial and error, so the above expression must exceed $V(L) = \Theta L$. This gives

$$\frac{1-w}{1-\beta}\frac{b-1}{b-2}L > \Theta L.$$

Multiplying both sides by $\beta(b-2)/L$ gives

$$\frac{\beta}{1-\beta}(b-1)(1-w) > \beta(b-2)\Theta.$$

Plugging this into equation (7) shows that $y^* > w^{-\frac{1}{b-1}}$, so inequality (12) holds, and there is more learning than in the myopic case. **Q.E.D.**

PROPOSITION 3: In the boom phase, asset prices increase more quickly when β is bigger.

PROOF: Firm value is a linear function, $V_t = V(x_{t-1}) = \Theta x_{t-1}$. In the boom phase, $x_t = y^* x_{t-1}$. From Proposition 1 we know that, when β is bigger, y^* is bigger, which means that in the boom phase, the $x_t's$ grow more quickly, so the $V_t's$ grow more quickly. **Q.E.D.**

PROPOSITION 4: The crash occurs sooner when β is bigger.

PROOF: By Proposition 2, when β is bigger, y^* is bigger. Since $b > 1$, it is obvious from equation (11) that $E(N)$ gets smaller. **Q.E.D.**

The more patient the investors, then, the more rapid the boom, but the sooner the bust. For example, lower interest rates encourage firms to invest in new projects, which accelerates reaching the capacity of a new technology. Then firm value drops. Expansionary monetary policy, by reducing interest rates, and so increasing the patience level of investors, accelerates the boom-bust cycle. Note, however, that expansionary monetary policy will not *cause* the boom-bust cycle in this model.

4.2 The influence of the wage rate w

After she learns A, the manager can always use the optimal and efficient input level $x = A$. This is the benefit of her experimentation. There are also costs involved. For all the periods before she learns A, she has not realized all the potential of the technology, so there is an opportunity cost of lost output. In period t, the quantity of the variable factor, $x_t = y^* x_{t-1}$, exceedes A, so she learns A. The oversupply of the variable factor is a cost to the firm because some of it is wasted. That cost is $w(x_t - A)$. The bigger the w, the higher this cost. The manager therefore tends to experiment less when w is big.

PROPOSITION 5: A higher wage rate hinders experimentation.

PROOF: **NOT DONE YET.**

In the myopic case, when $w = 1$, $x^* = L$, there is no experimentation; when $w = 0$, x^* could be infinitely big. These two extreme cases suggest that a higher wage rate makes the manager experiment more conservatively, as suggested by Proposition 5.

PROPOSITION 6: The higher the wage rate, the longer it takes the manager to learn the potential of the technology.

PROOF: Conservative experimentation means that it takes more time to learn A, so the expected number of periods until a crash increases. By Proposition 5, a bigger

w means y^* is smaller. By equation (11), this causes $E(N)$ to get bigger. **Q.E.D.**

PROPOSITION 7: The higher the wage rate, the smaller the firm value.

PROOF: When the wage rate w increases, two effects work simultaneously. One, which might be called "the direct effect," is that the cost to the firm for a given amount of input gets bigger. The other, "the indirect effect," is that the manager lowers the experimentation, as argued in Proposition 5. By the envelope theorem, at the optimal input level, only direct effect matters, so costs increase, and firm value decreases. **Q.E.D.**

4.3 The influence of the distribution parameter b

We assume the unknown potential, A, has a Pareto distribution. For bigger values of b, the probability density curve is steeper, and A is more closely clustered near L (Figure 4). This makes it easier to learn A.

However, when b gets bigger, the expected value of A, $E(A) = [(b-1)/(b-2)]L$, gets smaller. The manager will therefore experiment less. This effect means it takes longer to learn A. Numerical simulations suggest that the second effect dominates (see Table 1, last panel). That is, when b gets bigger, the expected number of periods until A is learned gets bigger, though at a decreasing rate. However, it is not clear whether this will always happen.

5. Concluding Remarks

We set up a dynamic infinite horizon model to study high-tech stock price volatility as a consequence of learning about the potential of a new technology. Without departing from the two standard economic assumptions that investors are rational and asset prices reflect fundamental values, our model generates a boom-bust cycle in stock prices. We thus provide an alternative to the popular bubble theory of recent asset price movements. The micro-foundation for our model is the process by which the manager learns about a new technology's unknown potential. Until the manager exhausts this potential, she does not know how high this potential can go. She therefore continually raises her expectation of this potential, and carries out more aggressive experimentation. This process goes on until the manager learns the actual potential. The firm's value then drops.

We derive the expected size of the crash and the expected number of periods before the crash. The influences of parameter values are also examined.

The adoption of a Pareto distribution for the unknown productivity and the Leontief production function makes the analysis convenient, but might limit the generality of the model. Studying different possible "learning technologies" might also be useful in future work.

NOTES

1. There are two broad categories of bubble theories: rational bubbles (Tirole, 1985, Allen, Morris, and Postlewaite, 1993, and Conlon, 2003) and irrational bubbles (Shiller, 2000, Abreu and Brunnermeier, 2003, and Scheinkman and Xiong, 2003). The rational bubble approach allows stock prices to diverge from fundamentals, but maintains the assumption that investors are rational. The irrational bubble approach assumes that investors' irrational behavior causes prices to differ from fundamentals.

2. Learning-by-doing models can be divided into models where productivity growth is a mechanical function of output versus Bayesian models, like the one here, where the random variables agents learn about are explicitly included in the model. Muth (1986) suggests that learning can result from a random search of input space coupled with a record of best practice results. Dorroh et al. (1994) suggests that results that have been regarded as learning by doing may merely result from expenditure of resources in the pursuit of production knowledge. Several other authors (see Levy, 1965, and Telser, 1982) also explain learning as the result of allocating resources to search or research and development.

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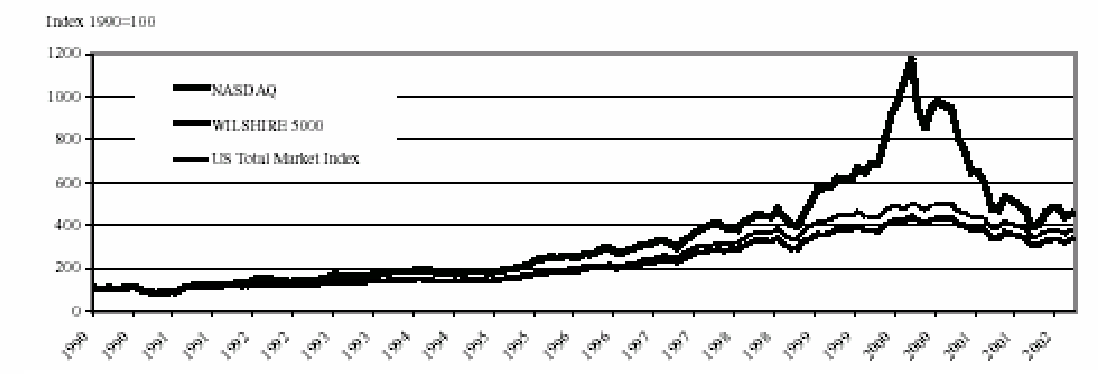
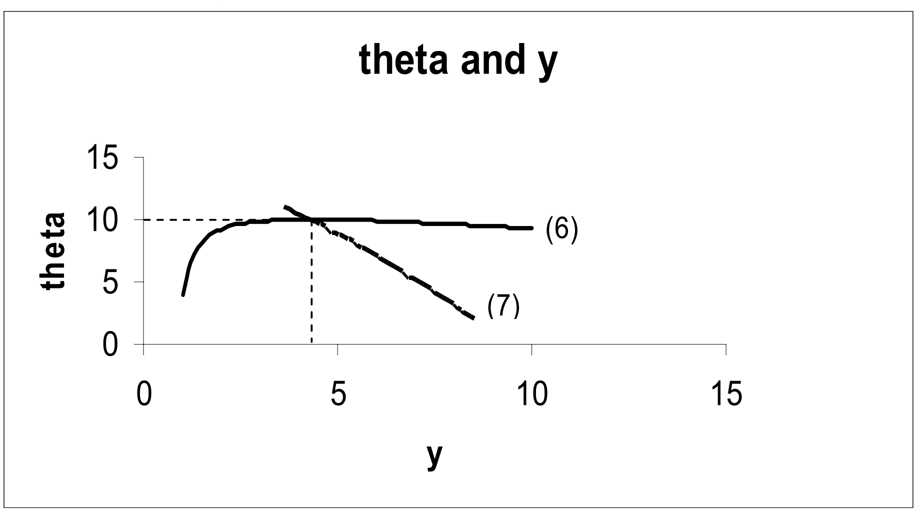
Figures and Tables

Figure 1 Stock Prices in the United States



Source: "Recent Experiences of Asset Price Bubbles (Comments by Ignazio Visco)," Figure 3.
Federal Reserve Bank of Chicago
Conference on Asset Price Bubbles, 22-24 April 2002

Figure 2 Graphs of equations (6) and (7)



w=0.2, b=4, beta=0.8
The crossing point is our solution.

Figure 3 Boom-busy Cycle

(w=0.8, b=4, beta=0.8, L=100, A=1000)

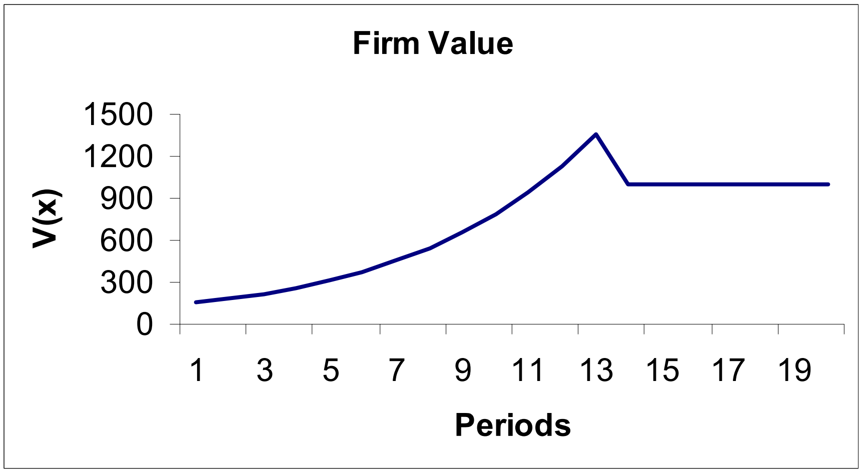


Figure 4 PDF of A with different b's (L=100)

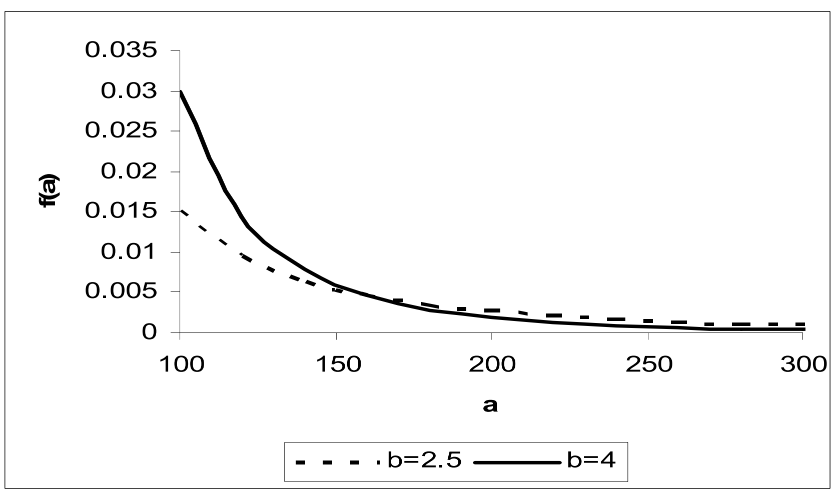


Table 1 Simulation Results

beta	w	b	min delta/V_t	max delta/V_t	E(delta)/V_t	E(N)
0.5	0.9	4	-0.056	0.006	-0.026	5.782
0.55	0.9	4	-0.065	0.001	-0.033	5.385
0.6	0.9	4	-0.076	-0.005	-0.042	4.997
0.65	0.9	4	-0.091	-0.014	-0.054	4.619
0.7	0.9	4	-0.108	-0.025	-0.069	4.254
0.75	0.9	4	-0.131	-0.040	-0.088	3.904
0.8	0.9	4	-0.158	-0.061	-0.113	3.572
0.85	0.9	4	-0.192	-0.087	-0.144	3.262
0.9	0.9	4	-0.232	-0.120	-0.181	2.977
0.95	**0.9**	**4**	**-0.280**	**-0.161**	**-0.226**	**2.719**
0.95	0.45	4	-0.318	0.068	-0.181	1.353
0.95	0.5	4	-0.316	0.035	-0.187	1.406
0.95	0.55	4	-0.314	0.005	-0.194	1.467
0.95	0.6	4	-0.312	-0.022	-0.200	1.536
0.95	0.65	4	-0.309	-0.048	-0.206	1.619
0.95	0.7	4	-0.306	-0.072	-0.212	1.720
0.95	0.75	4	-0.302	-0.096	-0.217	1.850
0.95	0.8	4	-0.297	-0.118	-0.221	2.024
0.95	0.85	4	-0.291	-0.140	-0.225	2.280
0.95	**0.9**	**4**	**-0.280**	**-0.161**	**-0.226**	**2.719**
0.95	0.9	2.2	-0.741	-0.535	-0.660	1.985
0.95	0.9	2.4	-0.623	-0.421	-0.539	2.212
0.95	0.9	2.6	-0.539	-0.349	-0.458	2.352
0.95	0.9	2.8	-0.476	-0.298	-0.399	2.450
0.95	0.9	3	-0.426	-0.261	-0.354	2.523
0.95	0.9	3.2	-0.385	-0.232	-0.318	2.580
0.95	0.9	3.4	-0.352	-0.209	-0.289	2.625
0.95	0.9	3.6	-0.324	-0.190	-0.264	2.662
0.95	0.9	3.8	-0.300	-0.174	-0.244	2.693
0.95	**0.9**	**4**	**-0.280**	**-0.161**	**-0.226**	**2.719**