Cognitive Tool #1 - Pythagorean Theorem Proofs (Java)
http://www.ies.co.jp/math/java/geo/pythagoras.html
This tool is a collection of different proofs of the Pythagorean Theorem. It allows the user to manipulate the size of the triangle to show that the Theorem holds over all right triangles. In some proofs, the user uses transformations and rotations to show the equality of $a^2 + b^2$ and $c^2$. In others, the user chooses how to break up $a^2$ and $b^2$ and then uses transformations to show that these pieces can also make up $c^2$. It is a good tool for students who may not quite understand what the Pythagorean Theorem truly means. It will help my students to understand the principles behind the Theorem and also to more accurately apply it in the future.

**Pythagorean Theorem**

<table>
<thead>
<tr>
<th>Pythagorean Theorem (1)</th>
<th>Pythagoras Theorem(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>Pythagorean Theorem (3)</td>
<td>Pythagorean Theorem (4)</td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td>Pythagoras Theorem(5)</td>
<td>Pythagorean Theorem(6)</td>
</tr>
<tr>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
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<tr>
<td>Pythagorean Theorem(7)</td>
<td>Pythagoras Theorem(8)</td>
</tr>
<tr>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
</tr>
<tr>
<td>Pythagorean Theorem(9)</td>
<td>Hypococrates' lunar</td>
</tr>
<tr>
<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
</tr>
</tbody>
</table>
How to use this applet

- Drag the red point.
- Press "Define" button.
- Drag five pieces to fit in the square below.

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Cognitive Tool #2 - Excel Spreadsheet

This tool is designed at allowing the students to discover the relationship between the angles of a triangle and the squares of its sides. Students will see that for all acute triangles $a^2 + b^2 > c^2$, and for all obtuse triangles $a^2 + b^2 < c^2$. They should also discover that the closer the largest angles is to 90 degrees, the smaller the difference between $a^2+b^2$ and $c^2$ will be. The tool will also allow the students to design their own Excel formulas to see the differences here and become better acquainted with the Theorem in general.

(when the students enter)
### Table: Triangle Properties

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>60</td>
<td>50</td>
<td>25</td>
<td>-25</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>90</td>
<td>25</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>136</td>
<td>74</td>
<td>100</td>
<td>26</td>
</tr>
</tbody>
</table>

(an example student creation)
The Lesson

Day 1 is designed at ingraining the meaning of the Pythagorean Theorem into the kids. We will talk about how we might use the theorem to solve the problem and also why we can use the theorem to create a solution for a given problem. Students should become more comfortable with the theorem and applying it to solve problems.

Day 2 is designed at using the theorem to understand the relationship between the sides of a triangle and its angles. Students should understand which triangles the theorem fails for and how it fails. The students will be asked why they think the theorem fails for acute and obtuse triangles and why it fails differently for the two groups.

The lesson is aimed at seventh grade students in a math classroom. The students already have an understanding of the Pythagorean Theorem and have already used it to solve abstract problems. The students are still struggling with correctly using the theorem and in which instances they can use it. These lessons are aimed at reducing these problems and giving the students some time to directly interact with the theorem in two different contexts.

The lessons are both about 90 minutes long.
Day 1: Proving the Pythagorean Theorem

90 minutes

OBJECTIVES
We will prove the Pythagorean Theorem.
We will use the Pythagorean Theorem to solve the Baseball Diamond Problem (below).

MATERIALS
computers (one per student)
internet connection for all computers
Mozilla Firefox web browser
Java support for all computers
LCD projector for main (teacher) computer

PROCEDURES
When the students come into the classroom, the teacher will have the computers on and the web browsers will already be turned to the Pythagorean Proof web page.

(10 minutes) Class Discussion of the Pythagorean Theorem. The class will discuss what they already know about the Pythagorean Theorem. What does it mean? How can we use it? In what contexts outside the classroom might it be helpful.
"What does Theorem mean?" The teacher should elicit ideas from the students.
"Theorems are ideas or theories that have been proven to be true over many different instances. The Pythagorean Theorem works for ANY right triangle. There are tons of right triangles out there, but the Pythagorean Theorem says that for any, 'the square of a leg plus the square of the other leg is the square of the hypotenuse.' Take a look at your web browser. Here are some different proofs of the Pythagorean Theorem. Try and use the first one in several different ways to show that for whatever sized right triangle, the Pythagorean Theorem holds."

(50 minutes) Independent Cognitive Tool. Students should use the tool to prove the Pythagorean Theorem. They should explore each of the different proofs. For proofs that allow the user to change the size of the triangle, they should use each several times to determine if they can find a configuration that the Theorem does not work for.

(20 minutes) Presentation of the Baseball Diamond Problem. The teacher will present the problem as such:
"On a major league baseball diamond, the distance between home plate and first base is 90 feet and the distance between first and second base is 90 ft. Sometimes, when a player steals second base, he steals it when the pitcher has just pitched the ball. If the catcher catches the pitch, he may try and throw out the stealing player. If he does, he needs to throw the baseball from home plate to second base. In about the same time how much farther has the baseball traveled than the stealing player?"

The teacher will lead a discussion about how the class might discover the answer to this problem. If the discussion turns to using the Pythagorean Theorem, or if the discussion stalls with no mention of it, the teacher should ask the students to turn to the second tab on their web browser. The Students will read and study the problem. The teacher will conduct a second discussion in which we talk about the application of the Pythagorean Theorem to solve the problem.

CLOSURE
(10 minutes) "Can we use any of these proofs to prove that the Pythagorean Theorem will work for our
baseball diamond problem?"
Have the student that volunteers, come to the main computer with projector and show how he/she could use one of the proofs to show that the solution for the Baseball Diamond Problem is valid.

ASSESSMENT
The student will be expected to solve the Baseball Diamond Problem for homework. He/she will also be expected to write a one paragraph explanation of why the Pythagorean Theorem allowed him/her to solve the problem. What does the Theorem say and why is it true?
Day 2: Does it Work for Other Triangles?

75 minutes

OBJECTIVES
We will evaluate the Pythagorean Theorem using acute, right, and obtuse triangles.

MATERIALS
computers (one per student)
Microsoft Office or OpenOffice.org for each computer (Excel)
LCD projector for main (teacher) computer
flash drive
worksheet of several different triangles
protractors
rulers (centimeters)

PROCEDURES
When the students come into the classroom, the teacher will have the computers on and the spreadsheet program will already be on and maximized.

(5 minutes) Class Discussion of the Pythagorean Theorem. What do we know about the theorem? What kinds of triangles does it work for?

(60 minutes) Independent Cognitive Tool. Students will measure the lengths of all three sides of each triangle using a ruler (centimeters). Side A should be the smallest side, b the middle, and c the largest side of the triangle. They will also use a protractor to measure the angles of the triangles. The side lengths and the measure of the largest angle will be placed in a spreadsheet. They should collect their data for all triangles and put it in the spreadsheet.

When the students have collected all of their data they should create a way for the spreadsheet to determine if the sum of the squares of sides a and b is equal to the square of side c. This information should be presented in a printout of the spreadsheet and also in a printout of a graph or chart that visually represents the data and uses the Pythagorean Theorem to classify the triangles into three groups. The spreadsheet and chart along with a description of the project and the findings of it will serve as the assessment for the day.

CLOSURE
(10 minutes) Class Discussion. "What did you discover today?" Class should focus on the way in which the Theorem grouped the triangles into three different groups and what other criteria might allow us to group the triangles into the same three groups (acute, right, and obtuse triangles). One student will show us his/her visual representation of the data.

ASSESSMENT
The student will complete the spreadsheet along with creating several more columns for evaluating the difference between a^2 + b^2 and c^2 over the different triangles. He/she will also create a chart to visually show the groupings of the triangles using this criteria and how the difference relates to the size of the largest angle of the triangles. He/she will also write a brief description of what he/she did and why they think that the Pythagorean Theorem grouped the triangles in such a way.