Biot Lecture, Columbia University,
Department of Civil Engineering and Engineering Mechanics,
19 October 2005

*Biot Poromechanics in
Earthquake and Faulting Phenomena*

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Examples, pore fluid interactions with deformation and failure in earth materials

• **Large scale pore fluid processes in the seismogenic lithosphere:**

  Elevated pore pressure in fault zones.

  → Solitary waves of pore pressure change; pulses of fluid outflow.
  Non-volcanic seismic tremors downdip of seismogenic subduction interface.

• **Poroelasticity in crustal materials:**

  → Post-seismic poroelastic deformation.
  Poroelastic effects in earthquake interactions, stress transfers, aftershocks.

  → Alteration of effective stress during rupture by dissimilarity of properties across fault plane

• **Fluid interactions with shear rupture in fault gouge:**

  → Extreme shear localization.
  Dilatant stabilization of slip; relation to lack of shallow EQ nucleation?

  → Thermal pressurization of pore fluid by shear heating.
  Partial melting of fault zone, pseudotachylytes.

• **Other:**

  Sediment liquefaction in cyclic shearing.
  Landslide to debris-flow transition.

  …
Linear Poroelasticity
**Poroelasticity**

**Linear quasi-static version** -- Terzaghi (1923), Biot (1935, 1941, 1973)  
(notation here of Rice & Cleary, 1976)

(total; no need to introduce separate stresses on solid and fluid)

\[
\sigma_{ij} = (K - \frac{2}{3}G)\delta_{ij}\varepsilon_{kk} + 2G\varepsilon_{ij} - \alpha\delta_{ij}p
\]

\[K = \text{drained (} p = \text{const.) bulk modulus, } G = \text{shear modulus, } \alpha = \text{new Biot poroelastic constant}\]

\[m - m_o = \rho_f \alpha(\varepsilon_{kk} + \frac{\alpha}{K_u - K} p)\]

[consistent with \(\sigma_{ij}d\varepsilon_{ij} + pd(m / \rho_f) = \text{perfect differential}]\]

\[K_u = \text{another new poroelastic constant = undrained (} m = \text{const.) bulk modulus; } K_u > K\]

\[\varepsilon_{ij} = \text{strains, } m = \frac{\text{fluid mass}}{\text{unit (reference) volume}}\]

Elastic isotropic constitutive relations:
Governing field equations:

- infinitesimal geometry changes of solid
- \( \nabla \cdot \sigma = 0 \) (equilibrium, neglecting body force or describing perturbation from gravity-loaded state)
- \( \nabla \cdot \boldsymbol{q} + \frac{\partial m}{\partial t} = 0 \) (\( \boldsymbol{q} \) = mass flux of fluid relative to solid)
- \( \boldsymbol{q} = -\frac{\rho_f k}{\eta_f} \nabla p \) (Darcy, neglecting body force or describing perturbation from gravity-loaded state)
- \( \varepsilon = \text{sym}(\nabla \mathbf{u}) \) (\( \mathbf{u} \) = solid displacement)

Final set of pde’s (in \( \mathbf{u} \) and \( p \)):

\[
(K + \frac{1}{3} G) \nabla(\nabla \cdot \mathbf{u}) + G \nabla^2 \mathbf{u} - \alpha \nabla p = 0 , \quad (\alpha_h \nabla^2 - \frac{\partial}{\partial t})(\nabla \cdot \mathbf{u} + \frac{\alpha}{K_u - K} p) = 0 \quad \left( \alpha_h = \frac{k}{\eta_f} \frac{(K_u - K)(K + 4G/3)}{\alpha^2(K_u + 4G/3)} \right)
\]

Other poroelastic parameters:

\[
\nu_u = \frac{\nu + (1-2\nu)(K_u - K)/3K_u}{1 - (1-2\nu)(K_u - K)/3K_u} , \quad B = \frac{K_u - K}{\alpha K_u}
\]

[ \nu_u = undrained \( (m = \text{const.}) \) Poisson ratio; \( \nu = \text{drained} \ (p = \text{const.}) \) Poisson ratio; \( 1/2 > \nu_u > \nu \) ]

[ \ B = \text{Skempton coefficient}, \ (dp)_{\text{undrained}} = -B \ d(\sigma_{kk})/3; \ 0 \leq B < 1 \]

For homogeneous and interconnected fluid phase, and solid phase which responds isotropically to pure pressure with same bulk modulus \( K_s \) at all points:

\[
\alpha = 1 - \frac{K}{K_s} , \quad K_u = K + \frac{\alpha^2 K_s K_f}{nK_s + (\alpha - n)K_f} \quad \left( n = \frac{m}{\rho_f} = \frac{\text{fluid volume}}{\text{unit (reference) volume}} \approx \text{porosity} \right)
\]
TABLE II. — Values of the drained and undrained Poisson ratios, \( v \) and \( v_a \), and of various ratios of undrained to drained stiffness based upon them. The intact-rock results are from a tabulation by Rice and Cleary [99]. The results given in terms of the crack density parameter \( N r^2 \) are from a tabulation by Rice and Rudnicki [123], based on self-consistent model calculations by O'Connell and Budiansky [132] for a solid with Poisson ratio 0.25 containing \( N \) cracklike pores of radius \( r \) per unit volume.

<table>
<thead>
<tr>
<th>Intact rock type [99]</th>
<th>( v )</th>
<th>( v_a )</th>
<th>( \frac{1 - v}{1 - v_a} )</th>
<th>( \frac{\xi}{\xi_a} ) for axisymmetric narrow ellipse</th>
<th>( \frac{\xi}{\xi_a} ) for sphere</th>
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</thead>
<tbody>
<tr>
<td>Charcoal granite</td>
<td>0.27</td>
<td>0.30</td>
<td>1.04</td>
<td></td>
<td></td>
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<tr>
<td>Westerly granite</td>
<td>0.25</td>
<td>0.34</td>
<td>1.14</td>
<td></td>
<td></td>
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<tr>
<td>Ruhr sandstone</td>
<td>0.12</td>
<td>0.31</td>
<td>1.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Berea sandstone</td>
<td>0.20</td>
<td>0.33</td>
<td>1.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clay soil</td>
<td>0.12</td>
<td>0.50</td>
<td>1.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( N r^2 ) [123, 132]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
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<td>1.25</td>
</tr>
<tr>
<td>0.4</td>
<td>0.08</td>
<td>0.41</td>
<td>1.56</td>
<td>1.29</td>
<td>1.39</td>
</tr>
</tbody>
</table>
**Example:** Dislocation suddenly introduced, at \( t = 0 \), along semi-infinite fault

(Rice & Cleary, 1976, generalizing Nur and Booker solution for incompressible constituents):

\[
\delta = \text{slip, uniform}
\]

![Diagram with symbols and equations]

\[
F(\infty) = 1 \quad \text{(undrained limit,} \ t = 0^+ \text{, or} \ x \to \infty) , \quad F(0) = \frac{1 - \nu(1 - \nu)}{1 - \nu} < 1 \quad \text{(drained limit,} \ t \to \infty \text{, or} \ x \to 0^+) \]

**Elementary model of a finite fault** (slipped and then locked):

- **Compressed**, causes uplift near surface, and \( P \) increase.
- **Expanded**, causes subsidence near surface, and \( P \) decrease.

But both effects diminish with time (due to undrained to drained transition).

**Subsidence** → **Uplift**

**Uplift** → **Subsidence**

Contributes to aftershocks along rupture zone?
Co-seismic well level increases (black circles) and decreases (white circles)

Figure 1 Map showing the location of the two June 2000 (moment magnitude $M_w = 6.5$) earthquakes in south Iceland. Inset, the approximate location of the mid-Atlantic plate boundary in Iceland, which has a full spreading rate of 1.94 cm yr$^{-1}$ (ref. 25). The south Iceland seismic zone (SISZ) is an east–west-oriented left-lateral transform zone (small blue box marks area shown in detail). The map shows mainshock epicentres (red stars) and focal mechanisms (Harvard CMT$^{26}$), other earthquakes from 21 June to 31 December 2000 (green dots), mapped surface ruptures$^{22}$ (yellow lines), and model fault traces as estimated from an inversion of the coseismic GPS and InSAR data for fault geometry$^{11}$ (purple lines). Also shown is the 17 June coseismic water-level increase (black dots) and decrease (white dots) in geothermal wells in the area$^{30}$. Note water-level increase in compressional quadrants. Solid rectangle marks the area shown in Fig. 2a–d and the larger dashed rectangle the area covered in Fig. 2f.
Figure 3: Co-seismic and post-seismic water-level changes in geothermal wells in south Iceland. a, The 21 June co-seismic water-level increase (black dots) and decrease (white dots) as well as the predicted co-seismic pore-pressure change at 0.5 km depth (normalized by $B \Delta p/B = -\Delta \sigma_{\text{gr}}/3$). b, Post-seismic water-level changes after 21 June and predicted post-seismic pore-pressure changes ($\Delta p/B = +\Delta \sigma_{\text{gr}}/3$). Water-level changes in labelled wells are shown as functions of time in Fig. 4a.

Figure 4: Water-level recoveries and aftershock decay in south Iceland. a, Observed peak-to-peak LOS displacement (red symbols) across the 17 June fault in several interferograms indicates that the post-seismic deformation transient lasted 2 months. Water-level changes (blue lines) show similar recovery of 1–2 months (well locations are shown in Fig. 3b). Well GS (squares) is located in co-seismic compressional quadrants and experiences post-seismic water-level decline, whereas wells KH (triangles), FL (diamonds) and HR (circles) are located in co-seismic extensional quadrants and exhibit post-seismic water-level increase. Note the rapid water-level recovery of well HR, located just south of the 17 June fault. b, Aftershock decay after the 17 June earthquake showing much longer timescale. Extrapolation of the ongoing aftershock sequence within 5 km of the 17 June fault predicts a total duration of 3.5 years. Same analysis for the 21 June fault-zone predicts aftershock duration of 3.3 years.
Figure 2 Synthetic aperture radar interferograms (InSAR) showing observed and simulated post-seismic deformation in south Iceland. The interferograms are unwrapped\textsuperscript{77}, and show ground displacements in the line of sight (LOS) towards the European radar satellite ERS-2 (range decrease is shown positive). The measurement is most sensitive to vertical ground motion, as the LOS vectors are roughly [east, north, up] = [0.38, −0.11, 0.92] for image a acquired from a descending orbit and [−0.41, −0.11, 0.90] for image f from an ascending orbit. The post-seismic interferograms span a, 19 June to 24 July (track 95, frame 2313) and f, 24 June to 29 July (track 173, frame 1297). In a, coseismic deformation caused by the 21 June earthquake, 17 km to the west, has been removed using a fault-slip model\textsuperscript{77}. Also shown are simulated interferograms of post-seismic deformation using poro-elastic (b), right-lateral aftserslip (c), and visco-elastic (d) models. The poro-elastic model prediction, b, is calculated using an undrained Poisson’s ratio $\nu_u$ of 0.31 and a drained Poisson’s ratio $\nu$ of 0.27 (refs 15, 24, 28), as well as the coseismic slip distribution for the two earthquakes\textsuperscript{71}. The aftserslip model assumes up to 50 cm of right-lateral slip centred at 8 km depth. The visco-elastic model has a 10-km-thick elastic crust overlaysing a visco-elastic half-space with viscosity of $10^{17}$ Pa s, chosen to match the time duration of the observed deformation transient. In e and g the average of LOS displacements profiles (between dashed lines) for interferograms in a and f are shown, as well as for two other interferograms (not shown) spanning the time intervals from 21 June (22 h after the 21 June earthquake) to 30 August (ascending track 130, frame 1287), and 12 August to 16 September (descending track 367, frame 2313). The response from a layered poro-elastic model (green dashed line), with drainage in only the top 1.5 km of the crust (see Methods section), shows similar response as the homogenous model (black dashed line and b) when the difference of the Poisson’s ratios $(\nu_u - \nu)$ is scaled (here by 2).
Mature crustal fault zones

Some observations on their structure, composition, and hydrologic properties
Fig. 1. Generalized geologic map of the San Gabriel Mountains and vicinity, southern California. Stippled pattern represents crystalline rocks. Bold arrows indicate the study localities discussed in the text: Devil's Canyon (DC), Bear Creek (BC), and Punchbowl (P). Key: active trace of the San Andreas fault (SAF), San Gabriel fault (SGF), North Branch San Gabriel fault (NBSGF), South Branch San Gabriel fault (SBSGF), Punchbowl fault (PF), Sierra Madre-Cucamonga thrust (SMCT), San Antonio fault (SA), Vincent thrust (VT), Fenner fault (FF), Soledad fault (SF), San Francisquito fault (SFF), Ridge basin (RB), Soledad basin (SB) and Punchbowl basin (PB).
Internal Structure of Principal Faults of the North Branch San Gabriel Fault

1) Undeformed Host Rock
2) Damaged Host Rock
3) Foliated Zone
4) Central ultracataclasite layer

Fault Zone

Fault Core

30-100 m
(Damage ≈ highly cracked rock. Zone with macro faults or fractures extends ~ 10x further.)

1-10 m
(Sometimes described as foliated gouge, or for some faults, simply as gouge.)

10s-100s mm
(But principal failure surface is much thinner, typically < 1-5 mm!)

Fig. 2. Schematic section across the North Branch San Gabriel fault zone illustrating position of the structural zones of the fault. The diagram is not to scale.

Prominent slip surface (pss) is located in the center of the layer and identified by the black arrows. (Exhumed from 2-4 km depth. Total slip ≈ 44 km. "Several km" of slip in earthquakes on the pss.)

Punchbowl Fault prominent slip surface

Figure 2. Photomosaic of entire thin section 1b (see Figure 1) under cross-polarized light showing the pss between the cohesive and less cohesive yellow-brown ultracataclasite. The pss is distinguished by uniform birefringence within a narrow planar band having sharp boundaries. Note the presence of relict pss and truncation of layering at the pss.
Fig. 1. (a) The Median Tectonic Line (MTL) and adjacent tectonic units in SW Japan. ISTL denotes the Itoigawa–Shizuoka Tectonic Line. (b) Geological map of the Median Tectonic Line near Taikide, western Mie Prefecture, including structural data on the deformation fabrics around the Median Tectonic Line. The stereograms are lower-hemisphere equal-area stereographic projections.
Median Tectonic Line Fault, Japan

Fig. 11. Sketch summary of the main elements of permeability structure across the Median Tectonic Line. (a) Summary of the structural zones; (b) summary permeability data distribution for different confining pressures (stated at the base, with * denoting data from the deconfining path), for 20 MPa pore pressure, given the mapped distribution of fault rocks shown in Figs. 1–3. Note that the distance axis is logarithmic in both directions away from the Ryoke/Sambagawa contact. ‘Cmt’ and ‘Inc’ denote cemented and incohesive foliated cataclasites, respectively, and ‘Cg’ denotes crenulated gouge.
(Wibberley and Shimamoto, *JSG*, 2003) permeability of clay gouge containing the central slip zone, Median Tectonic Line Fault, Japan
Physics of Fault Zone Processes during Seismic Slips

Questions:

• How does shear stress ($\tau$) vary with slip ($\delta$) during earthquakes? Focus is on weakening during rapid, large slip $\delta$ on mature faults, i.e., $\delta \gg 0.01-0.1$ mm (the slip range at which earthquakes are thought to nucleate, according to rate & state concepts and lab-based properties).

• What are the physical mechanism of weakening during slip? Suggested here: Primary mechanisms are
  • Thermal pressurization of pore fluid, and only this one discussed today
  • Flash heating at highly stressed frictional contacts.
Both seem to be important.
At sufficiently large slip, others mechanisms may become important too:
  • Melting if large enough normal stress (deeper slip),
  • Gel formation in lithologies of high silica content.

• What fracture energy ($G$) is implied by the $\tau$ vs. $\delta$ relation? Important because we can thereby test any proposed $\tau$ vs. $\delta$ against seismic constraints on $G$. 
Background for theoretical modeling of stress vs. slip relation:

Field and lab observations, exposures of mature, highly slipped fault zones:

- Slip in individual events is localized to a thin shear zone \((h < 1-5 \text{ mm})\) within a finely granulated \((\text{ultracataclastic, possibly clayey})\) fault core that is of order 10s to 100s mm thickness, with low permeability (estimated \(k \sim 10^{-20} \text{ m}^2\) at mid-seismogenic depths)

  [that despite the existence of much wider \((\sim 1-100 \text{ m})\) damage zones with granulation, pervasive cracking and/or minor faulting]

Hypotheses:

- Earthquake failure occurs in a water-saturated fault zone (a porous granular material in the shallow to middle crust).

- It has material properties (permeability, porosity, poroelastic moduli) like those inferred from lab studies of fault materials from the Median Tectonic Line (MTL), Nojima and Hanaore faults in Japan.

  [locations for which relatively complete data exists]
Governing equations, 1-space-dimension shearing field, constant normal stress \( \sigma_n \):

- **Constitutive relation (for shear):**
  \[ \frac{\partial u}{\partial y} = \frac{\mathcal{G}}{\mu} + \mathcal{G} (\mathcal{G} = \text{inelastic shear rate}) \]

**Friction law:** \( \tau = f \cdot (\sigma_n - p) \) if \( \mathcal{G} \neq 0 \) (\( \mathcal{G} \geq 0 \)).

- **Equations of motion (= equilibrium equations):**
  \[ \frac{\partial \sigma_{yy}}{\partial y} = 0 , \quad \frac{\partial \sigma_{yx}}{\partial y} = 0 \]
  \[ \Rightarrow \sigma_n = -\sigma_{yy} = \text{const.} , \quad \tau = \sigma_{yx} = \tau(t) \]

**Thermal pressurization of pore fluid**

- **Energy equation:**
  \[ \tau \mathcal{G} = \rho_c \frac{\partial T}{\partial t} + \frac{\partial q_h}{\partial y} , \quad q_h = -k \frac{\partial T}{\partial y} \]
  \( \rho_c \approx 2.7 \text{ MPa}/^\circ\text{C} ; \quad \alpha_{th} = \frac{K}{\rho_c} \approx 0.5-0.7 \text{ mm}^2/\text{s}. \)

- **Fluid mass conservation:**
  \[ \frac{\partial m}{\partial t} + \frac{\partial q_f}{\partial y} = 0 , \quad q_f = -\frac{\rho_f k}{\eta_f} \frac{\partial p}{\partial y} \Rightarrow \frac{\partial p}{\partial t} = \Lambda \frac{\partial T}{\partial t} - \frac{1}{\beta} \frac{\partial n^{pl}}{\partial t} + \alpha_{hy} \frac{\partial^2 p}{\partial y^2} ; \quad \alpha_{hy} = \frac{k}{\eta_f \beta} \]
  \( \Lambda \approx 0.3-1.0 \text{ (MPa}/^\circ\text{C}) , \quad \beta \equiv n(\beta_f + \beta_n) = 5.5-30 \times 10^{-11}/\text{Pa}; \)
  \( \beta_f, \beta_n = \text{fluid compressibility, pore space expansivity}. \)
Some calculations:

\[ m = \rho_f n \quad n = \frac{\text{volume of pore-fluid}}{\text{unit (reference) vol. of porous medium}} \]

\[ \dot{n} = \dot{\chi}_f n + \rho_f \ddot{n} = \dot{\chi}_f n + \rho_f (\dot{n}^e + \dot{n}^p) \]

\[ \dot{\rho}_f = \rho_f \beta_f \ddot{n} - \rho_f \lambda_f \ddot{n}^e, \quad \ddot{n}^e = n \beta_n \ddot{n} + n \lambda_n \ddot{n} \]

\[ \frac{\dot{m}}{\rho_f} = \beta (\ddot{n} - \Lambda \ddot{n}^p) + \ddot{n}^p, \]

where \( \beta = n(\beta_f + \beta_n) \) and \( \Lambda = \frac{\lambda_f - \lambda_n}{\beta_f + \beta_n} \).

Diffusivity \( \alpha_{hy} = \frac{k}{\eta_f \beta} \)
A perspective on shear localization in fluid-infiltrated granular media

Assume that all inelastic dilatancy $\Delta n_{pl}$ is over at small shear.

Then the governing equations for $p$ and $T$ are:

$$\frac{1}{\rho c} \tau(t) \& (y, t) = \frac{\partial T}{\partial t} - \alpha_{th} \frac{\partial^2 T}{\partial y^2}$$

$$\Lambda \frac{\partial T}{\partial t} = \frac{\partial p}{\partial t} - \alpha_{hy} \frac{\partial^2 p}{\partial y^2} .$$

$$\tau(t) = f[\sigma_n - p(y, t)] \text{ if } \& (y, t) \neq 0$$

**Question**: What type of solutions exist, if we assume that $f = \text{constant}$? Answer:

**Either**

(i) $p(y, t)$ is spatially uniform, $p(y, t) = p(t)$, $\Rightarrow T(y, t) = T(t)$, $\Rightarrow \& (y, t) = \& (t)$; i.e., no fluid flow (undrained), no heat flow (adiabatic), homogeneous strain (too idealized to be realistic, and has been proven to be unstable to perturbations [Rice and Rudnicki, 2005]),

or

(ii) $\& (y, t) = 0$ except at the isolated position(s) $y$ where $p(y, t) = \text{global maximum}$; $\& (y, t) = V(t)\delta_{\text{Dirac}}(y)$ for global max at $y = 0$ [V(t) = slip rate].
**Slip on a plane** at slip rate $V$ (Thickness $h$ of shearing layer assumed small compared to boundary layers where $p$ and $T$ increase):

- In $|y| > 0$, \( \frac{\partial T}{\partial t} = \alpha_{th} \frac{\partial^2 T}{\partial y^2} \) and \( \frac{\partial p}{\partial t} - \Lambda \frac{\partial T}{\partial t} = \alpha_{hy} \frac{\partial^2 p}{\partial y^2} \).

- On $y = 0^\pm$, \( q_h = -K \frac{\partial T}{\partial y} = \pm \frac{f(\sigma_n - p)V}{2} \); \( q_f = -\frac{\rho f k}{\eta_f} \frac{\partial p}{\partial y} = 0 \).

- Assumes all dilatancy $\Delta n^{pl}$ (distributed) is over at small slip $[p_{amb} \rightarrow p = p_{amb} - \frac{\Delta n^{pl}}{\beta}]$.

**Simple solution**: For $V \equiv d\delta/dt = \text{constant}$, and $f = \text{constant}$, we solve for the fields $T(y,t)$ and $p(y,t)$, and hence $p(0,t)$, to evaluate

\[ \tau = \tau(\delta) = f(\sigma_n - p(0,t)) \quad (\text{where slip} \ \delta = Vt) : \]

\[ \tau(\delta) = f(\sigma_n - p_o) \exp \left( \frac{\delta}{L^*} \right) \text{erfc} \left( \sqrt{\frac{\delta}{L^*}} \right), \]

where \( L^* = \frac{4}{f^2} \left( \frac{\rho c}{\Lambda} \right)^2 \left( \sqrt{\alpha_{hy}} + \sqrt{\alpha_{th}} \right)^2 \).  

[Mase & Smith, 1987; Rice, 2005]
Slip on a Plane, stress vs. slip ($V = \text{const.}$):

\[ \frac{\tau}{f(\sigma_n - p_o)} = \exp \left( \frac{\delta}{L^*} \right) \text{erfc} \left( \frac{\sqrt{\delta}}{\sqrt{L^*}} \right); \quad L^* = \frac{4}{f^2} \left( \frac{\rho c}{\Lambda} \right)^2 \left( \sqrt{\alpha_{hy}} + \sqrt{\alpha_{th}} \right)^2. \]

Note apparent multi-scale nature of the slip-weakening; no well-defined $D_c$:

On slip plane, $T - T_{amb} = \Delta T_{\text{max}} \left[ 1 - \frac{\tau}{f(\sigma_n - p_o)} \right]$, $\Delta T_{\text{max}} = \sqrt{\frac{f^2 V L^*}{4 \alpha_{th}}} \frac{\sigma_n - p_o}{\rho c}$. 

\[ L^* = 4 \text{ mm} \quad \rightarrow \quad 0.4 \text{ mm} \quad 4 \text{ mm} \quad 40 \text{ mm} \quad 0.4 \text{ m} \]
\[ L^* = 30 \text{ mm} \quad \rightarrow \quad 3 \text{ mm} \quad 30 \text{ mm} \quad 0.3 \text{ m} \quad 3 \text{ m} \]
How large is $L^*$?

$$L^* = \frac{4}{f^2} \left( \frac{\rho c}{\Lambda} \right)^2 \left( \sqrt{\alpha_{hy}} + \sqrt{\alpha_{th}} \right)^2 \frac{2}{V}.$$ 

Evaluations for 7 km depth, typical centroidal depth of crustal rupture zone; 
\(\sigma_n = \) overburden = 196 MPa, \(p_o = p_{amb} = \) hydrostatic = 70 MPa, \(T_{amb} = 210^\circ C\):

Part of $L^*$ based on poro-thermo-elastic properties of fault gouge [Rice, 2005, to \textit{JGR}]:

\[
\left( \frac{\rho c}{\Lambda} \right)^2 \left( \sqrt{\alpha_{hy}} + \sqrt{\alpha_{th}} \right)^2 \approx \begin{cases} 
60 \text{ mm}^2/\text{s (low end)} & \Rightarrow L^* \approx 4 \text{ mm, if } V = 1 \text{ m/s and } f = 0.25. \\
450 \text{ mm}^2/\text{s (high end)} & \Rightarrow L^* \approx 30 \text{ mm, if } V = 1 \text{ m/s and } f = 0.25, \\
& \text{or } L^* \approx 50 \text{ mm, if } V = 1 \text{ m/s and } f = 0.20.
\end{cases}
\]

Accounting approximately for \textit{damage} at the rupture front and during subsequent shear, 
\(k_{dmg}^d = 5-10 \, k, \, \beta_{dmg}^d = 1.5-2 \, \beta_d\)

- \(V = 1 \text{ m/s}\) is the average ratio of \textit{slip} to \textit{slip duration} at a point, for the 7 slip inversions discussed in [Heaton, 1990], range is 0.56 to 1.75 m/s, average is 1.06 m/s).

- \(f = 0.25\) represents effects of \textit{flash heating}, like in \textit{high-speed friction experiments} [Tullis and Goldsby, 2003; Prakash, 2004].
Stresses off the main fault plane: Why we have to worry about effects of fresh damage on $k$ and $\beta$


$\Rightarrow$ denotes parameter estimated from seismic slip inversion, Heaton (1990) -- $L$, $\delta$ and $v_r$

Locked-in slip $\Delta u_x = \delta$

Slipped, and now locked again

Currently slipping

Locked, not yet slipped

To formulate as an elasticity problem for a steady-state field, dependent on $x - v_r t$ and $y$, we specify:

$v_r$, $L$, $\delta$, $\frac{\tau_{\text{peak}}}{-\sigma_{yy}}$, $\frac{\tau_{\text{res}}}{\tau_{\text{peak}}}$, $\Psi$ and one of $\frac{R}{L}$ or $\frac{\tau_0 - \tau_{\text{res}}}{\tau_{\text{peak}} - \tau_{\text{res}}}$.

From Heaton (1990) $f_{\text{peak}} \approx 0.6$ e.g., 0.2
Procedure:

(1) Solve the 2D elasticity problem, calculate stresses $\sigma_{\alpha\beta}$.

(2) Check if the $\sigma_{\alpha\beta}$ would cause Coulomb shear failure $[\tau > (-\sigma_n)\times\tan(31^\circ)]$ on some Orientation, or cause tensile failure $[\sigma_2 > 0]$.
Poroelastic effects included; Skempton $B = 0.6 \ [\Delta p = -B \Delta(\sigma_{11} + \sigma_{22} + \sigma_{33})/3 \ ]$

For $R/L = 0.1$, and $\tau_r / \tau_p = 0.2$ (nearly complete strength loss)

YELLOW = shear failure  RED = tensile failure

Scale length in plots:

$(R_0^*)_{avg} = 20-30 \ m,$

$R_0^* = 1-70 \ m,$

fitting model to Heaton (1990) earthquake set, assuming $f_p = 0.6$ and hydrostatic initial pore pressure.
Table 1: Properties of the ultracataclastic, clayey gouge containing the principal slip surface of the Median Tectonic Line fault zone, Japan. Results from Wibberley [2002] and private communication [2003], for pore compressibility parameter $-\partial n / \partial \sigma_c$ (where $\beta_d$ is drained bulk compressibility of the porous medium and $\beta_s$ is compressibility of its solid grains) and for $n$ (= porosity; precisely, the void volume per unit reference state volume of the porous aggregate). Results from Wibberley and Shimamoto [2003] in their Figure 8.b.ii provide permeability $k$; the value shown here at effective confining stress $\sigma_c - p = 126$ MPa corresponds to their results for isotropic virgin consolidation to that confining stress. Their gouge was instead consolidated further, to $\sigma_c - p = 180$ MPa, and then studied at various states of unloading (unloading and reloading are then approximately reversible) to provide the results in their Figure 8.b.ii. Permeability values $k$ shown here are estimated values for an unloading curve starting at 126 MPa, assuming that at any given effective stress, the ratios of permeability along that curve, to those along the actually documented curve for unloading from 180 MPa, are in the same 1.91 ratio as the ratio of the virgin consolidation $k$ (0.65 x 10^{-20} m^2) at 126 MPa to the $k$ (0.34 x 10^{-20} m^2) at that same 126 MPa along the unloading curve from virgin consolidation to 180 MPa. Numbers in brackets are interpolated or extrapolated. Parameters $\beta_n$ and $\lambda_n$ enter an expression of type $dn = n(\beta_n dp + \lambda_n dT)$ characterizing effect of variation of pore pressure $p$ and temperature $T$ at various external constraints. Explanation of superscripts on $\beta_n$ and $\lambda_n$: "v" is for variation at fixed confining stress $\sigma_c$; "el" and "dmg" are for variation at fixed fault-normal stress $\sigma_n$ and zero fault-parallel strains, with "el" for elastic response of fault wall and "dmg" to approximately represent a damaged wall state with inelastic response for which, for the values shown here, $\beta_d$ has been doubled, i.e., $\beta_d^{dmg} = 2.0 \beta_d$. For that damaged state, the permeability $k^{dmg}$ has been increased to 10 times $k$. For calculations of the table, the following values have been assumed: $\beta_s = 1.6 \times 10^{-11} / \text{Pa}$; solid grains volumetric thermal expansion $\lambda_s = 2.4 \times 10^{-5} \degree \text{C}$ ($\lambda_n^{v} = \lambda_n^{dmg} = \lambda_s$); drained Poisson ratio of aggregate $\nu_d = 0.20$.
### Models considered:

<table>
<thead>
<tr>
<th></th>
<th>Intact Elastic Walls</th>
<th>Highly Damaged Walls</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average on $p$ and $T$</td>
<td>Average on $p-T$ path</td>
</tr>
<tr>
<td>Common parameters assumed for all models:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Specific heat of fault gouge [L], $c_p$ (MPa/°C)</td>
<td>2.7</td>
<td>2.7</td>
</tr>
<tr>
<td>Starting porosity [W], $n$</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Friction coefficient [flash heating, see text], $f$</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Slip rate [see text], $v$ (m/s)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Normal stress, $\sigma_n$ (MPa)</td>
<td>196</td>
<td>196</td>
</tr>
</tbody>
</table>

**Path ranges used for property evaluations:**

- Pore fluid pressure range $p_{amb}$, $p_{high}$ (MPa): 70, 70, 133, 70, 70, 133
- Effective stress range $\sigma_n - p_{amb}$, $\sigma_n - p_{high}$ (MPa): 126, 126, 126, 126, 126, 126
- Temperature range $T_{amb}$, $T_{high}$ (°C): 210, 210, 210, 210, 210, 210

**Material properties (averages over path ranges):**

<table>
<thead>
<tr>
<th>Property</th>
<th>$\theta_0$ (mm$^2$/s)</th>
<th>$\lambda_f$ (10$^{-3}$/°C)</th>
<th>$\beta_f$ (10$^{-9}$/Pa)</th>
<th>$\beta_p$ (10$^{-9}$/Pa)</th>
<th>$\eta_f$ (10$^{-4}$/Pa·s)</th>
<th>$k$ (10$^{-20}$ m$^2$)</th>
<th>$A$ (MPa/°C)</th>
<th>$\alpha_{hy}$ (mm$^2$/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal diffusivity [VS]</td>
<td>0.70</td>
<td>0.65</td>
<td>0.70</td>
<td>0.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluid thermal expansivity [B]</td>
<td>1.08</td>
<td>1.21</td>
<td>1.08</td>
<td>2.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pore space thermal expansivity, $\lambda_p$ (10$^{-3}$/°C)</td>
<td>$-$0.19</td>
<td>$-$0.18</td>
<td>0.02</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluid compressibility [B]</td>
<td>0.64</td>
<td>0.74</td>
<td>0.64</td>
<td>4.47</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Pore space pressure expansivity [W]</td>
<td>0.65</td>
<td>0.77</td>
<td>2.49</td>
<td>2.95</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluid viscosity [K] / [T]</td>
<td>1.48</td>
<td>1.26</td>
<td>1.48</td>
<td>0.77</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Permeability [WS], $k$ (10$^{-20}$ m$^2$)</td>
<td>0.65</td>
<td>1.38</td>
<td>6.5</td>
<td>13.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Resulting material properties:**

- Undrained pressurization factor, $A$ (MPa/°C): 0.98, 0.92, 0.34, 0.31
- Hydraulic diffusivity, $\alpha_{hy}$ (mm$^2$/s): 0.86, 1.81, 3.52, 6.04

**Resulting parameters of slip-on-plane model:**

- Weakening length parameter, $L_w$ (mm): 1.51, 2.55, 29.8, 49.5
- Maximum possible temperature rise, $\Delta_T_{max}$ (°C): 271, 366, 1,200, 1,840
- Maximum possible $T_{max}$ ($^{\circ}$C) (for $T_{amb}$ = 210°C): 481, 576, 1,410, 2,050

---

Table 2: Assumed and resulting parameters of the slip-on-plane model, to represent a mature fault surface at 7 km depth, at normal stress of 196 MPa, ambient pore pressure of 70 MPa, and ambient temperature of 210 °C. "Intact Elastic Walls" models use laboratory-constrained data for permeability $k$ and pore space pressure expansivity $\beta_p = \beta^{mg}_p$ of undisturbed gouge, and "Highly Damaged Walls" models account approximately but arbitrarily for gouge damage at the rupture front, and during slip and thermal pressurization, by using a differently defined pore space pressure expansivity $\beta_p = \beta^{mg}_p$ and increasing $k$ by 10× and $\beta_p$ by 2× the laboratory-constrained values; $k$ and the two $\beta_p$ measures are assumed to vary only with $\sigma_n - p$. Fluid and other $T$ or $T$-$p$ dependent properties are evaluated as averages along straight line paths from $p_{amb}$, $T_{amb}$ to $p_{high}$, $T_{high}$. The "high" values are set to ambient values in the 1st and 3rd columns. Results there are used to approximately estimate the $p_{high}$, $T_{high}$ for the 2nd and 4th columns, respectively, as rough spatial averages, over the part of the wall that is actively participating in the heat and mass transfer, at the stage when $p$ at the fault has been elevated to $\sigma_n$. Codes: B: Burnham et al. [1969]; K: Keenan et al. [1978]; L: Leachnirich [1980]; T: Todreide [1972]; V: Vosteen and Schellschmidt [2003]; W: Wibberley [2002] and private communication [2003]; WS: Wibberley and Shimamoto [2003]. Fluid properties estimated from B, K and T with the collaboration of Dr. A. Rempel.
General definition of fracture energy associated with slip weakening on a fault:

\[ G \equiv \int_{0}^{\delta_{\text{large}}} [\tau(\delta') - \tau_{\text{res}}] d\delta' \quad (\tau(\delta_{\text{large}}) \approx \tau_{\text{res}}) . \]

When a uniform residual level \( \tau_{\text{res}} \) has not been reached at maximum slip \( \delta \), a consistent definition is:

\[ G = G(\delta) \equiv \int_{0}^{\delta} [\tau(\delta') - \tau(\delta)] d\delta' . \]

Does not include contributions to \( G \) from inelastic deformation near the fault plane. How large?
Slip on a Plane, energy release rate

\[ G = G(\delta) = \int_0^\delta [\tau(\delta') - \tau(\delta)] d\delta' = f(\sigma_n - p_o) L^* \left[ \exp \left( \frac{\delta}{L^*} \right) \text{erfc} \left( \sqrt{\frac{\delta}{L^*}} \right) \left( 1 - \frac{\delta}{L^*} \right) - 1 + 2 \sqrt{\frac{\delta}{\pi L^*}} \right] \]

For \( \frac{\delta}{L^*} \gg 1 \), \( G \rightarrow 2(\sigma_n - p_o) \left( \frac{\rho c}{\Lambda} \right) \left( \sqrt{\frac{\alpha_{th} t}{\pi}} + \sqrt{\frac{\alpha_{hy} t}{\pi}} \right) \) (indep. of \( f \) and \( V \) -- \( t \) = slip duration)
Seismic estimates of fracture energy \((G)\):

**Method A Use of seismic slip inversion results for large sets of earthquakes:**

A.1: Fit of seismic slip inversion results from Heaton (PEPI, 1990) to a steady state, self-healing, slip pulse model (Rice, Sammis and Parsons, BSSA, 2005),

\[ G = \int_0^\delta (\tau(\delta') - \tau(\delta)) d\delta' \]

A.2: Tinti, Spudich and Cocco (JGR, in press 2005), use of kinematic slip \((\delta)\) inversions, smoothed, to get stress \((\tau)\) histories too, then use of

Fault along Earth’s surface

View onto fault plane during rupture:

previously slipped, now re-locked; slip = \(\delta\)
currently slipping
not yet slipped
Fracture energies $G$ and slips $\delta$, large earthquakes (arranged in order of slip magnitude)

<table>
<thead>
<tr>
<th>Event</th>
<th>$M_o$</th>
<th>$l$</th>
<th>$w$</th>
<th>$\delta$</th>
<th>$G$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Michoacan 1985 (M=8.1)</td>
<td>1,500</td>
<td>150</td>
<td>120</td>
<td>2.8</td>
<td>6.6</td>
<td>[1]</td>
</tr>
<tr>
<td>Landers 1992 (M=7.1)</td>
<td>56</td>
<td>60</td>
<td>14</td>
<td>2.2</td>
<td>5.0</td>
<td>[3]</td>
</tr>
<tr>
<td>Landers 1992 (M=7.3)</td>
<td>97</td>
<td>79</td>
<td>15</td>
<td>2.2</td>
<td>17.4</td>
<td>[2a]</td>
</tr>
<tr>
<td>San Fernando 1971 (M=6.5)</td>
<td>7</td>
<td>12</td>
<td>14</td>
<td>1.4</td>
<td>6.9</td>
<td>[1]</td>
</tr>
<tr>
<td>Northridge 1994 (M=6.7)</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>0.99</td>
<td>5.2</td>
<td>[2b]</td>
</tr>
<tr>
<td>Borah Peak 1983 (M=7.3)</td>
<td>23</td>
<td>40</td>
<td>20</td>
<td>0.96</td>
<td>2.9</td>
<td>[1]</td>
</tr>
<tr>
<td>Tottori 2000 (M=6.8)</td>
<td>13</td>
<td>29</td>
<td>18</td>
<td>0.80</td>
<td>3.7</td>
<td>[2c]</td>
</tr>
<tr>
<td>Kobe 1995 (M=6.9)</td>
<td>22</td>
<td>48</td>
<td>20</td>
<td>0.78</td>
<td>1.5</td>
<td>[4]</td>
</tr>
<tr>
<td>Kobe 1995 (M=6.9)</td>
<td>24</td>
<td>60</td>
<td>20</td>
<td>0.62</td>
<td>1.7</td>
<td>[2d]</td>
</tr>
<tr>
<td>Imperial Valley 1979 (M=6.5)</td>
<td>5</td>
<td>30</td>
<td>10</td>
<td>0.56</td>
<td>1.3</td>
<td>[1]</td>
</tr>
<tr>
<td>Imperial Valley 1979 (M=6.6)</td>
<td>7.7</td>
<td>35</td>
<td>12</td>
<td>0.6</td>
<td>0.81</td>
<td>[5]</td>
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<tr>
<td>Imperial Valley 1979 (M=6.6)</td>
<td>8.6</td>
<td>42</td>
<td>11</td>
<td>0.6</td>
<td>1.8</td>
<td>[2e]</td>
</tr>
<tr>
<td>Morgan Hill 1984 (M=6.2)</td>
<td>2.1</td>
<td>20</td>
<td>8</td>
<td>0.44</td>
<td>2.0</td>
<td>[1]</td>
</tr>
<tr>
<td>Morgan Hill 1984 (M=6.3)</td>
<td>2.7</td>
<td>30</td>
<td>10</td>
<td>0.26</td>
<td>2.0</td>
<td>[6]</td>
</tr>
<tr>
<td>Morgan Hill 1984 (M=6.3)</td>
<td>2.6</td>
<td>30</td>
<td>10</td>
<td>0.25</td>
<td>1.4</td>
<td>[2f]</td>
</tr>
<tr>
<td>Colfiorito 1997 (M=5.9)</td>
<td>0.71</td>
<td>10</td>
<td>7</td>
<td>0.38</td>
<td>0.83</td>
<td>[2g]</td>
</tr>
<tr>
<td>N. Palm Springs 1986 (M=6.0)</td>
<td>1.8</td>
<td>18</td>
<td>10</td>
<td>0.33</td>
<td>0.15</td>
<td>[1]</td>
</tr>
<tr>
<td>Coyote Lake 1979 (M=5.9)</td>
<td>0.35</td>
<td>6</td>
<td>6</td>
<td>0.32</td>
<td>0.57</td>
<td>[1]</td>
</tr>
</tbody>
</table>

[1] Rice, Sammis and Parsons [2005] based on slip inversions by Heaton [1990]. The $G$ values are averages of $G_{min}$ and $G_{max}$ ($= 2 G_{min}$) of Rice et al. [2005]; i.e., $G = 1.5 G_{min} = 0.75 G_{max}$.
Method B (Abercrombie and Rice, *GJI*, 2005),  
*Use of radiated energy, moment, and source area (hence stress drop and slip)*:  

**Notation:**

\[ E_s = \text{radiated seismic energy} \left( \int_S \int_0^\infty \rho(du/dt)^2 \, dt \, dS \right) \]

\[ A = \text{rupture area (from corner frequency or duration)} \]

\[ \delta = \text{final slip (from moment } M_o = \mu \delta A \text{)} \]

\[ \delta' = \text{variable slip as event develops} \]

\[ \tau_0 = \text{initial shear stress} \]

\[ \tau_1 = \text{final static shear stress (stress drop } \tau_0 - \tau_1 \propto \mu \delta / \sqrt{A} \text{)} \]

\[ \tau(\delta) [= \tau_{\text{dyn}}] = \text{stress in last increment of dynamic slip} \]

**Energy balance:**

\[ \left( \frac{\tau_0 + \tau_1}{2} \right) A = \left( \int_0^\delta \tau(\delta') \, d\delta' \right) A + E_s = \left( G + \tau_{\text{dyn}} \delta \right) A + E_s \]

\[ \text{since } G = \int_0^\delta [\tau(\delta') - \tau(\delta)] \, d\delta' = \int_0^\delta [\tau(\delta') - \tau_{\text{dyn}}] \, d\delta' \]

\[ \Rightarrow \quad \frac{\tau_0 - \tau_1}{2} \delta = G + (\tau_{\text{dyn}} - \tau_1) \delta + \frac{E_s}{A} \text{, or} \]

\[ G' \equiv G + (\tau_{\text{dyn}} - \tau_1) \delta = \left[ (\tau_0 - \tau_1) - \frac{2 \mu E_s}{M_o} \right] \delta / 2 \text{; } G' \approx G \text{ and } G' = G \text{ if } \tau_{\text{dyn}} = \tau_1 \]

[We find, with the Madariaga (1979) estimate of \( \tau_0 - \tau_1 \), that \( \mu E_s / M_o \approx 0.1(\tau_0 - \tau_1) \).]
Comparison, predictions of $G$ from the slip on a plane model with seismic estimates, for:

- The large earthquake data set for $G$ from seismic slip inversions for 12 events (shown as ovals here, one symbol per event), and

- A data set for $G'$ for small and large events based on radiated energy, moment, and seismic source dimension [Abercrombie & Rice, 2005]; $G' \approx G$, and $G' = G$ if stress during last increments of slip = final static stress after rupture (no overshoot/undershoot).

\[(\text{Rice, 2005, to } JGR)\]
Configurational stability of spatially uniform, adiabatic, undrained, shear:
(Motivation: Why do zones of localized slip have the thickness that they do?)

Governing equations for shearing velocity \(V(y, t)\), shear stress \(\tau(y, t)\), pore pressure \(p(y, t)\), and temperature \(T(y, t)\):

\[
\frac{\partial \tau}{\partial y} = 0 \ \text{(inertia irrelevant)}, \quad \sigma_n = \text{const.}
\]

\[
\tau = f(\frac{\partial V}{\partial y})(\sigma_n - p), \quad f'(\ldots) > 0
\]

\[
\frac{\tau}{\partial y} = \rho c \frac{\partial T}{\partial t} + \frac{\partial q_h}{\partial y}, \quad q_h = -\rho c \alpha_{th} \frac{\partial T}{\partial y}
\]

\[
\frac{\partial m}{\partial t} = \rho_f \beta \left( \frac{\partial p}{\partial t} - \Lambda \frac{\partial T}{\partial t} \right) = -\frac{\partial q_f}{\partial y}, \quad q_f = -\rho_f \beta \alpha_{hy} \frac{\partial p}{\partial y}
\]

The spatially uniform solution:

\[
V(y, t) = V_0(y) = \gamma_o y \quad (\gamma_o = \text{uniform shearing rate})
\]

\[
p(y, t) = p_0(t), \quad \tau(y, t) = \tau_0(t) = f(\gamma_o)[\sigma_n - p_0(t)]
\]

\[
\sigma_n - p_0(t) = [\sigma_n - p_0(0)] \exp(-H \gamma_o t) \quad [\text{call this } \bar{\sigma}_0(t)]
\]

where \(H \equiv \frac{f(\gamma_o)\Lambda}{\rho c} \approx 0.1-0.3 f(\gamma_o)\),

\[
T(y, t) = T_0(t) \quad \rho c dT_0(t)/dt = f(\gamma_o)\bar{\sigma}_0(t) \gamma_o
\]

Simple rate-strengthening friction model; approximately valid only in stable regions in which rupture cannot nucleate, but may propagate through (or in unstable regions that have shear-heated to a frictionally stable \(T\) range).

(Fuller rate-state description, with localization limiter, must be used in regions of unstable, rate-weakening, friction -- but localization is expected then.)
Linearized perturbation about time-dependent spatially uniform solution:

\[
V(y,t) = \gamma_0 y + V_1(y,t), \quad p(y,t) = p_0(t) + p_1(y,t),
T(y,t) = T_0(t) + T_1(y,t), \quad f = f(\gamma_0) + f'(\gamma_0)\partial V_1(y,t)/\partial y
\]

\[
\frac{\partial}{\partial y} \left( -f(\gamma_0)p_1 + f'(\gamma_0)\frac{\partial V_1}{\partial y} \bar{\sigma}_0(t) \right) = 0
\]

\[
f(\gamma_0)\bar{\sigma}_0(t)\frac{\partial V_1}{\partial y} - f(\gamma_0)p_1\gamma_o + f'(\gamma_0)\frac{\partial V_1}{\partial y} \bar{\sigma}_0(t)\gamma_o = \rho c \left( \frac{\partial T_1}{\partial t} - \alpha_{th} \frac{\partial^2 T_1}{\partial y^2} \right)
\]

\[
\frac{\partial p_1}{\partial t} - \Lambda \frac{\partial T_1}{\partial t} = \alpha_{hy} \frac{\partial^2 p_1}{\partial y^2}
\]

Nature of solution with spatial dependence \(\exp(2\pi iy/\lambda)\):

\[
\bar{\sigma}_0(t)\frac{\partial V_1(y,t)}{\partial y}, \quad p_1(y,t), \quad T_1(y,t) \propto \exp(st)\exp(2\pi iy/\lambda)
\]

\[
\bar{\sigma}_0(t) \propto \exp(-H\gamma_o t) \Rightarrow \frac{\partial V_1(y,t)}{\partial y} \propto \exp[(s + H\gamma_o)t] \exp(2\pi iy/\lambda)
\]

\(s = s(\lambda)\) satisfies:

\[
zH\gamma_o s = \left( s + \frac{4\pi^2\alpha_{th}}{\lambda^2} \right) \left( s + \frac{4\pi^2\alpha_{hy}}{\lambda^2} \right)
\]

where \(z = \frac{f(\gamma_o)}{\gamma_0 f'(\gamma_o)} = \frac{f}{a - b} \approx 0.6/0.015 = 40\)

Typically of order 20-60, in results of low shear-rate experiments.

Dynamic disk simulations [Chevoir et al., 05]

\(z > 1000\) for effective stress > 10 MPa
Condition for linear instability of flow profile \( \left( \frac{\partial V_1}{\partial y} \rightarrow \infty \right) \): 

\[
\text{Re}(s) + \frac{fA}{\rho c} \gamma_o > 0 \quad \Rightarrow \quad \lambda > \lambda_{cr} \equiv 2\pi \sqrt{\frac{(\alpha_{th} + \alpha_{hy})\rho c}{(z + 2)fA\gamma_o}}
\]

For shear of layer of thickness \( h \left( \gamma_o = \frac{V}{h} \right) \): 

\[
\lambda > \lambda_{cr} \equiv 2\pi \sqrt{\frac{(\alpha_{th} + \alpha_{hy})\rho ch}{(z + 2)fA V}}
\]

Comment: Near \( \lambda = \lambda_{cr} \), \( \text{Im}(s) \approx z \frac{fA}{\rho c} \frac{\sqrt{\alpha_{th}\alpha_{hy}}}{\alpha_{th} + \alpha_{hy}} \frac{V}{h} \) (oscillatory; unloading?)
Possible self-consistent estimate of shear layer thickness $h$ at large shear:

Set $\gamma_o = \frac{V}{h}$, $\lambda_{cr} \approx h \Rightarrow h \approx \frac{4\pi^2(\alpha_{th} + \alpha_{hy})\rho c}{(z + 2)f \Lambda V}$

Results (using $z = 40$, $V = 1$ m/s, $\alpha_{th} = 0.7$ mm$^2$/s, $\rho c = 2.7$ MPa/°C):

Low end ($\Lambda = 0.70$ MPa/°C, $\alpha_{hy} = 1.5$ mm$^2$/s): $h = 4.4$ µm / $f = 11$ µm (if $f = 0.4$).

High end ($\Lambda = 0.34$ MPa/°C, $\alpha_{hy} = 3.5$ mm$^2$/s): $h = 31$ µm / $f = 78$ µm (if $f = 0.4$).

Implications:

- Even with velocity strengthening [with large $z (= f / [\gamma df/d\gamma]$), e.g., of order $z > 10$], we must expect large shear strain to be confined to a thin zone, less than diffusion penetration distances of heat and fluid in moderate and larger events.

- Justifies use of model based on slip on a plane.

- Observed 1-5 mm deformed zone thickness in gouge may be a precursor thickness (i.e., $\lambda_{cr}$ based on an initial, broad $h$) not the thickness of the large shear zone.
Effective Normal Stress Alteration Due to Pore Pressure Changes Induced by Dynamic Slip Propagation on a Plane Between Dissimilar Materials

\[ \sigma_{yy}^0 = -\sigma_o \]

\[ \sigma_{xy}^0 = \tau_o \]

Dissimilar elastic materials "1" and "2" meet at slip surface

Damage fringes (also dissimilar)

(Rudnicki & Rice, 2005)
For steadily propagating mode II ruptures, in form slip = \( \delta = \delta(x - v t) \), on an interface between elastically dissimilar solids
(where \( v = \) rupture propagation speed, called \( v_r \) earlier), Weertman (1980) showed that

\[
\sigma_{xy}(x) = \sigma_{xy}^0 - \frac{\mu(v)}{\pi} \int_{-\infty}^{+\infty} \frac{d\delta(x')/dx'}{x - x'} dx'
\]

\[
\sigma_{yy}(x) = \sigma_{yy}^0 - \mu^*(v)d\delta(x)/dx
\]

Note that \(-d\delta/dx = V/\nu\) where \( V = \) slip rate. Much recent study of this case (Adams, 1995, 1998; Andrews & Ben-Zion, 1997; Harris & Day, 1997; Cochard & Rice, 2000; Ben-Zion & Huang, 2001; Brietzke & Ben-Zion, 2005).

When slip surface is bordered by damage fringes, of dissimilar poroelastic properties, which undergo undrained response except in a narrow boundary layer where the \( p \) discontinuity is reconciled, \( p \) and effective tensile stress on the slip plane are (Rudnicki & Rice, 2005)

\[
p(x) = p^0 - \frac{W(v)}{2} \frac{d\delta(x)}{dx},
\]

\[
\sigma_{yy}(x) + p(x) = (\sigma_{yy}^0 + p^0) - \left( \mu^*(v) + \frac{W(v)}{2} \right) \frac{d\delta(x)}{dx}
\]

Both \( W(v) \) and \( \mu^*(v) \) reverse sign when the materials on the two sides are interchanged; effective tension can either increase (destabilizing) or decrease (stabilizing) in different cases.
\[ \sigma_{yy}(x) + p(x) = (\sigma_{yy}^0 + p^0) - \left( \mu^*(v) + \frac{W(v)}{2} \right) \frac{d\delta(x)}{dx} \]

(tensile stress $\sigma_{yy}^0$ is negative)

Number on $W(v) / 2 \mu_2$ curves is ratio $k^+ \beta^+ / k^- \beta^-$

Case shown: $\mu_1 = 0.75 \mu_2$, $c_{s1} = 0.90 c_{s2}$, $B^+ = B^- = 0.6$
Consequence of strong dependence of permeability $k$ on effective stress:

*Lithostatic pore pressure gradients*

*Solitary waves of pressure increase*

Earth’s surface

Fault

σ

σ

pore pressure $p$

lithostatic stress $\sigma$

effective stress $\overline{\sigma} = \sigma - p$

solitary wave of $p$ increase

Upflow from deep fluid source
Mass conservation and Darcy flow in one-dimensional vertical seepage [with \( p = p(z,t) \)] in a fault zone channel of constant (irrelevant) width:

\[
\frac{\partial}{\partial z} \left[ -\frac{\rho_f k}{\eta_w} \left( \frac{\partial p}{\partial z} + \rho_f g \right) \right] + \frac{\partial m}{\partial t} = 0
\]

\[ k = k(\bar{\sigma}), \; m = m(\bar{\sigma}), \; \bar{\sigma} = \sigma - p \) (and \( \bar{\sigma} \) & \( \sigma \) positive in compression).

\( \sigma = \sigma(z) = \) total normal stress, satisfies (approximately) \( \frac{d\sigma}{dz} = -\rho_{tot} g \), so that

\[
\frac{\partial p(z,t)}{\partial z} + \rho_f g = -\frac{\partial \bar{\sigma}(z,t)}{\partial z} - \gamma \), where \( \gamma = (\rho_{tot} - \rho_f) g \).
\]

\[
\therefore \quad \frac{\partial}{\partial z} \left[ \frac{\rho_f k(\bar{\sigma})}{\eta_w} \left( \frac{\partial \bar{\sigma}}{\partial z} + \gamma \right) \right] + \frac{\partial m(\bar{\sigma})}{\partial t} = 0
\]

**Time-independent steady flow solution** on \( z < 0 \), with \( \bar{\sigma} = 0 \) at \( z = 0 \): \( \bar{\sigma} \) satisfies

\[
\quad k(\bar{\sigma}) \left( \frac{d\bar{\sigma}}{dz} + \gamma \right) = \text{constant} \quad \left( = \frac{\eta_w}{\rho_f} q_f \), where \( q_f \) is steady mass upflow rate \right)
\]

As \( z \to -\infty \), \( \bar{\sigma} \to \bar{\sigma}_o \), a constant, where \( \gamma k(\bar{\sigma}_o) = \frac{\eta_w}{\rho_f} q_f \); \( \int_{0}^{\bar{\sigma}(z)} \frac{k(\bar{\sigma}')d\bar{\sigma}'}{k(\bar{\sigma}') - k(\bar{\sigma}_o)} = -\gamma z \)

\[
\Rightarrow \quad \text{As} \; z \to -\infty, \quad \frac{dp}{dz} \to \frac{d\sigma}{dz} = -\rho_{tot} g \) (i.e., \( p \) gradient \( \to \) lithostatic).
Revil & Cathles, "Fluid transport by solitary waves along growing faults: A field example from the South Eugene Island Basin, Gulf of Mexico", *EPSL*, 2002

Fig. 3. Pore fluid overpressures (in MPa) in the downthrow and upthrow sides of the Red Fault system (Block 330). Three compartments can be observed on each side: C1 is the hydrostatic upper portion of the basin, C2 is a softly pressurized compartment, and C3 is a highly pressurized compartment. The boundaries S1 and S2 correspond to permeability barriers (shale and/or capillary barriers). Note the correlation between the location of S2 and the presence of the JD sand (the arrows surrounding the location of the JD sand on the upthrow side correspond to the shallowest and deepest depth of location of this sand). The filled circles correspond to direct pressure measurements made in the boreholes. The open circles represent mud weight data. The mud weight data correspond to the weight of the drilling mud used to drill the borehole and adjusted to compensate the fluid pressure of the formations in order to avoid any blow out of the borehole.
Solitary wave solutions on $-\infty < z < +\infty$, in form $\bar{\sigma} = \bar{\sigma}(t - z/V)$, to

$$\frac{\partial}{\partial z} \left[ \frac{\rho_f k(\bar{\sigma})}{\eta_w} \left( \frac{\partial \bar{\sigma}}{\partial z} + \gamma \right) \right] + \frac{\partial m(\bar{\sigma})}{\partial t} = 0 :$$

Both $k(\bar{\sigma})$ and $m(\bar{\sigma})$ decrease monotonically with increasing $\bar{\sigma}$, but $k(\bar{\sigma})$ is the more strongly varying,

in the precise sense that we assume

$$\frac{k(\bar{\sigma}) - k(\bar{\sigma}_2)}{k(\bar{\sigma}_1) - k(\bar{\sigma}_2)} < \frac{m(\bar{\sigma}) - m(\bar{\sigma}_2)}{m(\bar{\sigma}_1) - m(\bar{\sigma}_2)}$$

for all $\bar{\sigma}_1 < \bar{\sigma} < \bar{\sigma}_2$.

Example:

$k(\bar{\sigma}) \propto d^3$ and $m(\bar{\sigma}) \propto d$, where $d$ is a crack aperture, decreasing monotonically with increasing $\bar{\sigma}$:

$$\frac{k(\bar{\sigma}) - k(\bar{\sigma}_2)}{k(\bar{\sigma}_1) - k(\bar{\sigma}_2)} = \frac{d^3 - d_2^3}{d_1^3 - d_2^3} = \frac{d - d_2}{d_1 - d_2} \frac{(d^2 + dd_2 + d_2^2)}{d_1^2 + d_1d_2 + d_2^2} < \frac{d - d_2}{d_1 - d_2} = \frac{m(\bar{\sigma}) - m(\bar{\sigma}_2)}{m(\bar{\sigma}_1) - m(\bar{\sigma}_2)}$$

for all $d_1 > d > d_2$.

Result: Solutions exist if and only if $\bar{\sigma}_{\text{orig}} > \bar{\sigma}_{\text{final}}$ (i.e., corresponds to a $p$ increase)

and $V = \frac{\rho_f \gamma k(\bar{\sigma}_f) - k(\bar{\sigma}_o)}{\eta_w m(\bar{\sigma}_f) - m(\bar{\sigma}_o)} > 0$ (only upward propagation possible).
(Related study by Revil & Cathles, "Fluid transport by solitary waves along growing faults: A field example from the South Eugene Island Basin, Gulf of Mexico", *EPSL*, 2002)

**Figure 1** A fault caught in the act of burping. *a*, Map of the B-fault showing reflectivity from the fault plane in 1985. The area of highest reflectivity is circled in gold. *b*, Map of the B-fault reflectivity, as shown in *a*, but from 1992. The data extend over a slightly larger area than in *a*; however, the spatial perspective is identical. The area of highest reflectivity, circled in gold, is shifted roughly 1 km north-east in the updip direction relative to its location in 1985, as would be expected for a fluid pulse ascending the B-fault; this movement is depicted by the arrow in *a*. Also shown is the location of the A10ST well intersection, where exceptionally high fluid pressures were encountered while drilling into the B-fault zone in 1993.
Pulsed fluid flow at the toe of the Barbados accretionary wedge
[Henry, J. Geophys. Res., 105 (B11), 2000]

Figure 6. Permeability measurements from packer tests in cased holes, with open screen set at decollement level, compared with laboratory measurement on samples [Fisher and Zwart, 1997; Scretan et al., 1997; Zwart et al., 1997]. Lines connect data acquired on the same sample. Circles and squares are packer tests at Sites 948 and 949, respectively. Triangles are CORK data from submersible tests (Site 949). Open and solid symbols represent the pressure at the beginning and end of each permeability test, respectively.

Drillhole data here

Figure 14. Water flux at various locations along the decollement as a function of time. The flow pulse progressively widens but keeps a well defined leading edge. Equivalent permeability is given for a flow zone thickness of 40 m.
Melt as the pore fluid

Shear heating: Sometimes fault zones get hot enough that they (partially) melt

pictures only
Rapidly resolidified melt (now a glass, called pseudotachylyte) which was extruded -- one could say *hydraulically fractured* -- into cracks in material bordering a melting fault zone:

![Micrograph of pseudotachylyte](image)

**Fig. 1** - BSE photomicrograph of an area of glassy pseudotachylyte in a sample from Outer Hebrides Thrust of the Western Isles, northwest Scotland. Taken from Spray [1993].
Figure 1. Active faults in the Kobe-Osaka area (thin lines) including the surface trace of the seismic Nojima fault (thick line), and sampling locality of the fault rocks studied in this paper.

9 pseudotachylyte-generating events:

- Resolidified shear zones marked P1, …, P9; no or minimal overlap.
- All 9 shear zones fit within a 20 mm width! -- as well as 2 more not shown.
- Individual zones have $h < 2$ mm, often $< 1$ mm.

**Figure 3.** Photomicrograph of a thin section of thinly laminated fault rock. C: granitic cataclasite, G: fault gouge, and P: pseudotachylyte. Broken lines denote the boundaries between layers formed by different seismic slip events. A thick broken line in layer P6 indicates sinusoidal lamina.
Figure 9. Backscattered electron images of pseudotachylyte samples showing various degrees of melting. a: volume fraction $\phi$ of unmelted grains = 0.08, b: $\phi = 0.119$, c: $\phi = 0.415$, and d: $\phi > 0.482$. Dark gray grains: quartz, gray grains: plagioclase, and white grains: potassium feldspar (large) and Fe-rich spherules (small). Elongated vesicles are well developed in a and b.
Figure 16. Various temperature indices (above) and the estimated temperature of pseudotachylyte melt as a function of the volume fraction $\phi$ of unmelted grains (below). Melting started at about 750°C, and the maximum temperature reached 1280°C.
Examples shown, pore fluid interactions with deformation and failure in earth materials

• *Poroelasticity in crustal materials:*
  
  Post-seismic poroelastic deformation.
  Alteration of effective stress during rupture by dissimilarity of properties across fault plane

• *Fluid interactions with shear rupture in fault gouge:*
  
  Extreme shear localization.
  Thermal pressurization of pore fluid by shear heating, likely a primary weakening mechanism.

• *Large scale pore fluid processes in the seismogenic lithosphere:*
  
  Solitary waves of pore pressure change; pulsed fluid outflow.