Biot Medal Lecture 2003

Energy Approach to Poromechanics

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Energy Approach to Poromechanics

- rocks, soils
- concretes
- woods
- foams
- bones
- living tissues
- etc.

- drying and shrinkage
- wetting and swelling
- temperature rise and build-up of pressure
- hydraulic diffusion and subsidence
- freezing and spalling
- capillarity and cracking
- electro-osmosis and consolidation
- etc.

Outlines :

1. Revisiting the Energy Approach of Saturated Poroelasticity
2. Energy Approach to Partially Saturated Poroelasticity
3. From Poromechanics to the Mechanics of Colloids

Revisiting Biot's Energy Approach to Saturated Poroelasticity

Open system:

$m_{fl}\, d\Omega_0$ *fluid mass content*

Element of Porous material = *Solid Skeleton* ***sk*** + *Saturating fluid* ***fl***

Assuming no dissipation

$$\underbrace{\sigma_{ij}\, d\varepsilon_{ij}}_{\text{Strain work}} + \underbrace{\left(\underbrace{f_{fl}}_{\text{Free energy supplied by convection}} + \underbrace{\frac{p_{fl}}{\rho_{fl}}}_{\text{Work to make the mass enter the porous element}} \right) dm_{fl}}_{g_{fl}\, dm_{fl}} - \underbrace{dF}_{\text{Helmoltz Free energy infinitesimal change of the open system}} = 0$$

$$\overset{\uparrow}{g_{fl}}\, dm_{fl}$$

Gibbs potential

Separate equality for the skeleton

$$\sigma_{ij}\, d\varepsilon_{ij} + g_{fl}\, dm_{fl} - dF = 0$$

Mesoscopic information

$$dg_{fl} = \frac{dp_{fl}}{\rho_{fl}}$$

Fluid state equations

$$m_{fl} = \phi\, \rho_{fl}$$
$$F_{sk} = F - f_{fl}\, m_{fl}$$

Skeleton free energy

$$\underbrace{\sigma_{ij}\, d\varepsilon_{ij} + p_{fl}\, d\phi}_{\text{Strain work Related to the skeleton only}} - dF_{sk} = 0$$

Lagrangean porosity

Linear Poroelasticity and Poroplasticity (saturated)

$$\sigma_{ij}\, d\varepsilon_{ij} + p_{fl}\, d\phi - dF_{sk} \;=\; 0$$

State equations: $\sigma_{ij} = \dfrac{\partial F_{sk}}{\partial \varepsilon_{ij}} \quad p_{fl} = \dfrac{\partial F_{sk}}{\partial \phi}$

LINEAR POROELASTICITY

$$\boxed{\begin{gathered} \sigma = K\varepsilon - bp_{fl} \\ \phi - \phi_0 = p_{fl} / N + b\,\varepsilon \\ s_{ij} = 2G\, e_{ij} \end{gathered}}$$

POROPLASTICITY

$$\boxed{\begin{aligned} \varepsilon_{ij} &\rightarrow \varepsilon_{ij} - \varepsilon_{ij}^{p} \\ \phi &\rightarrow \phi - \phi^{p} \end{aligned}}$$

State equations

$$\boxed{f\left(\sigma_{ij}, p_{fl}\right) \le 0}$$

Plastic criterion

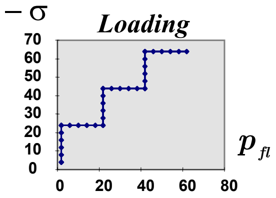
$$\boxed{d\varepsilon_{ij}^{p} = d\lambda \frac{\partial f}{\partial \sigma_{ij}}, \quad d\phi^{p} = d\lambda \frac{\partial f}{\partial p_{fl}}}$$

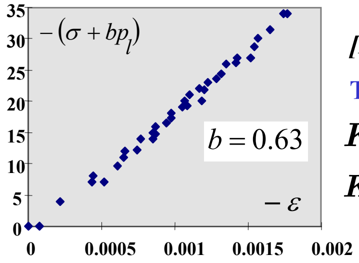
Flow rule

Biot's effective stress

$$\boxed{-\left(\sigma + bp_{l}\right) = -K\varepsilon}$$

Experimental validation ?



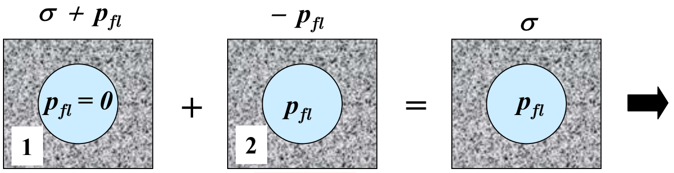


[Bouteca, 1994]

Tavel limestone

$K = 19500$ MPa

$K_s = 52700$ MPa



$$\boxed{0 \le \; b = 1 - \frac{K}{K_s} \le 1}$$

$$\boxed{\varepsilon_1 = \frac{\sigma + p_{fl}}{K}} \;+\; \boxed{\varepsilon_2 = -\frac{p_{fl}}{K_s}} \;=\; \boxed{\varepsilon = \frac{\sigma}{K} + p_{fl}\left(\frac{1}{K} - \frac{1}{K_s}\right)}$$

non-linear skeleton, linear solid matrix

$$\boxed{\varepsilon_1 = \frac{\sigma + p_{fl}}{K_{SEC}\left(\sigma + p_{fl}\right)}}$$

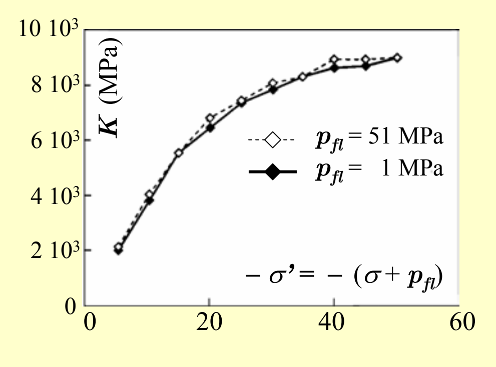
$$d\sigma = K\,d\varepsilon - b\,dp_{fl}$$

$$K = K\left(\sigma + p_{fl}\right)$$

$$b = 1 - K\left(\underbrace{\sigma + p_{fl}}\right)/K_s$$

Terzaghi's effective stress

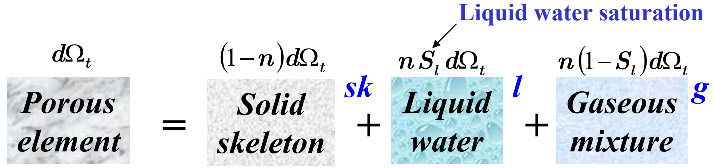
Dormieux et al., 2002



Bemer et al., 2001

Sandstone specimen

Extension of Energy Approach to Unsaturated Poroelasticity



Liquid water saturation

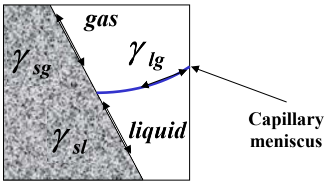
$d\Omega_t$ ***Porous element*** = $(1-n)d\Omega_t$ ***Solid skeleton*** ***sk*** + $n\,S_l\,d\Omega_t$ ***Liquid water*** ***l*** + $n(1-S_l)d\Omega_t$ ***Gaseous mixture*** ***g***

$$\underbrace{\sigma_{ij}}_{\text{Total stress}}\,d\varepsilon_{ij} + \underbrace{\left(S_l\,p_l + S_g\,p_g\right)}_{\text{Averaged pressure } p^*}d\phi - \phi\underbrace{p_c}_{p_c = p_g - p_l}dS_l - dF_{sk} = 0$$

Capillary pressure

$$F_{sk} = F_{solid} + \phi U$$

Skeleton energy — *Solid energy* — *Interface Energy*



$$\sigma_{ij} = \frac{\partial F_{sk}}{\partial \varepsilon_{ij}} \qquad p^* = \frac{\partial F_{sk}}{\partial \phi} \qquad -\phi p_c = \frac{\partial F_{sk}}{\partial S_l}$$

Unsaturated Poroelasticity: Equivalent Pore Pressure π

Coussy et Dangla, 2002

$$\phi p_c = -\frac{\partial F_{sk}}{\partial S_l}$$

$$p_c = p_c(S_l) \rightarrow F_{sk} = \overbrace{F_{solid}(\varepsilon_{ij},\phi)}^{\textit{Solid matrix}} + \overbrace{\phi U(S_l)}^{\textit{Interface energy}}$$

Energy separation

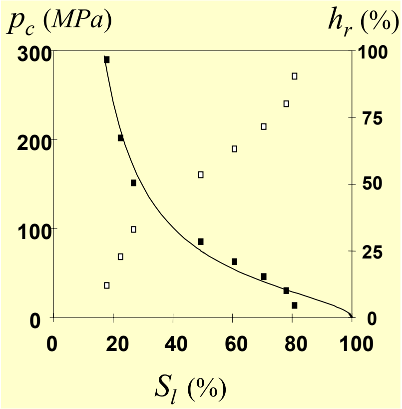
$$\sigma_{ij}\, d\varepsilon_{ij} + \pi\, d\phi - dF_{solid} = 0 \qquad \sigma_{ij} = \frac{\partial F_{solid}}{\partial \varepsilon_{ij}} \qquad \pi = \frac{\partial F_{solid}}{\partial \phi}$$

p^* **Averaged pore pressure (= Bishop's equivalent pore pressure)**

$$\pi = \overbrace{S_l\, p_l + S_g p_g} - U \equiv \textit{Equivalent pore pressure}$$

$$U = -\int_1^{S_l} p_c(s)\,ds$$

$$\sigma = K\varepsilon - b\pi \;!$$



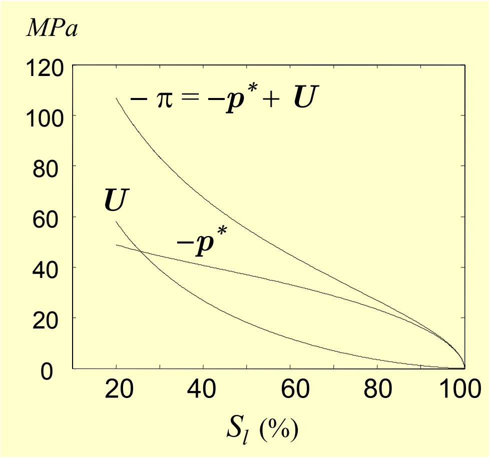
p_{atm} = 0.1MPa = p_g (neglected)

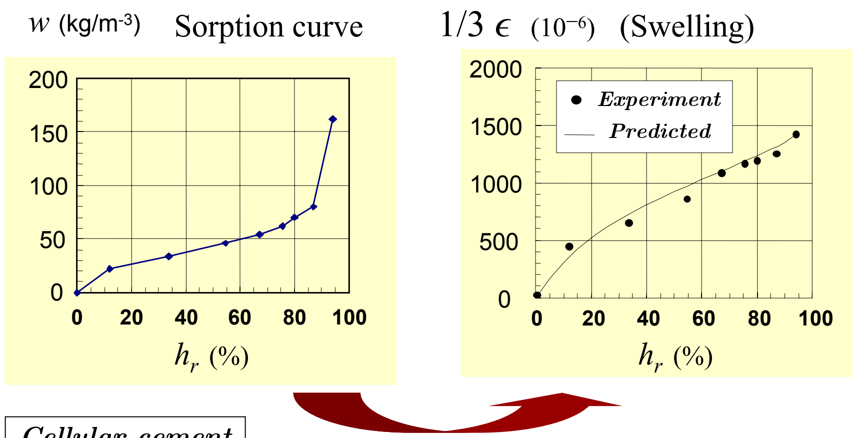
$\sigma_{ij} + p^*$ (*Bishop's effective stress*)

$\sigma_{ij} + \pi = \sigma_{ij} + p^* - U$ (*here*)

$$U = -\int_1^{S_l} p_c(s)\,ds$$

Cement paste

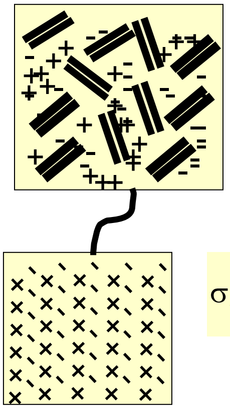




Cellular cement fiber composite
b = 0.79
Carmeliet, 1999

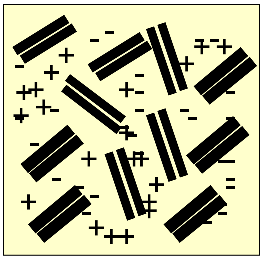
$$\epsilon = \frac{b}{nK} \times \frac{RT}{M_v} \times \int_{h_r}^{h_r^0} \frac{w}{h}\, dh \qquad \left(= \frac{b}{K}\pi \right)$$

From poromechanics to mechanics of colloids



At constant solution pressure and decreasing salt concentration:

SWELLING



$$\sigma_{ij}\, d\varepsilon_{ij} + p_w\, d\phi + p_e\, d\left(\frac{\phi}{A}\right) - dF_{sk} = 0 \qquad \frac{1}{A} = \frac{\rho_e}{\rho_e^{eff}} \quad A > 1$$

$$d\sigma = K\, d\varepsilon - dp_w - b_e\, dp_e$$

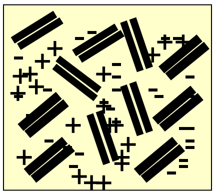
$$d\left(\frac{\phi}{A}\right) = b_e\, d\varepsilon + \frac{dp_e}{N}$$

$$p_e = 2RT\ c^{eff}$$

$$\ell = \sqrt{\frac{\varepsilon\varepsilon_0 RT}{2F^2 c^{eff}}}$$

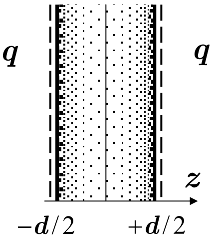
$$d\varepsilon = d\phi \ \Rightarrow\ b_e = \left(\frac{\partial(\phi/A)}{\partial\phi}\right)_{p_e}$$

From poromechanics to mechanics of colloids



$$s \times \frac{d}{2} = \phi$$

$$p_e = 2RT\, c^{eff}$$



$$d(\sigma + \overbrace{p_w + p_e}^{\textit{Solution pressure}}) = K\, d\varepsilon - dp_{sw}$$

$$p_{sw} = -\int (1 - b_e)\, dp_e$$

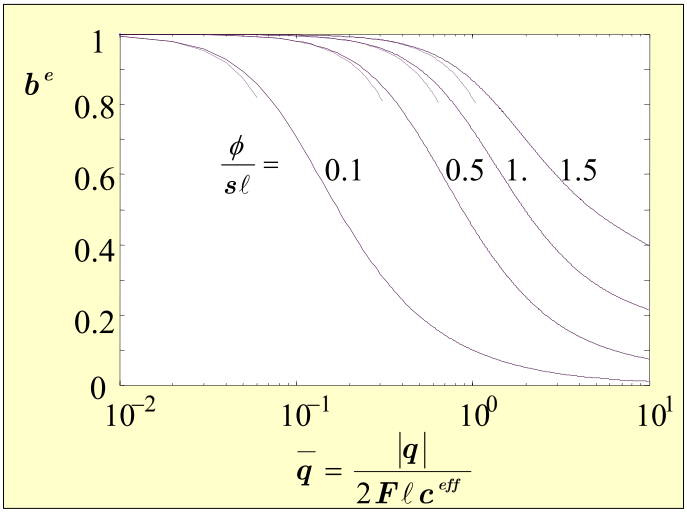
$$p_{sw} = RTc_+(0) + RTc_-(0) - p_e$$

$$b_e = \left(\frac{\partial(\phi / A)}{\partial \phi} \right)_{p_e} \quad \text{or} \quad b_e = \left(\frac{\partial p_{sw}}{\partial p_e} \right)_{\phi} + 1$$

Poromechanics

Porophysical Chemistry

$$d\sigma = K\, d\varepsilon - dp_w - b_e dp_e$$



C^{eff} *ranging from* 10^{-4} *to* 10^{-4} mol/l

ℓ *ranging from* 30 nm *to* 1 nm

$q = 0.01$ *to* $0.2\, C \times m^{-2}$

$$\ell = \sqrt{\frac{\varepsilon\varepsilon_0 RT}{2F^2 c^{eff}}}$$

As a conclusion build a bridge between …

Upscaling methods
thermodynamics

POROPHYSICAL
CHEMISTRY
Micro/Nano Scales

POROMECHANICS
Macro Scale

back analysis

Many Thanks To Many People
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