BIOT LECTURE: TWENTY-FIVE YEARS OF THE SLOW WAVE

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BRIEF HISTORY OF POROELASTICITY

1941 – Biot (quasi-statics, theory and analysis)

1944 – Frenkel (electroseismics and waves)

1951 – Gassmann (undrained behavior, theory)

1954 – Skempton (undrained behavior, experiments in soil)

1956 – Biot (waves, Lagrangian app., prediction of the slow wave)

1957 – Biot and Willis (coefficients from experiment)

1962 – Biot (reformulation of wave theory, Hamiltonian app.)

1976 – Rice and Cleary (quasi-statics, numerical methods)

1980 (Feb. 15) – Plona (slow wave first observed! but no theory!!)

1980 (Aug. 15) – Berryman (effective mass and "structure factor")

1980 (August) – Brown (elect. tortuosity and "structure factor")

1980 (Dec. 15) – Johnson (liquid He, 4th sound, and the slow wave)

OUTLINE

- What is the slow wave? Why is it important?
- Methods of measuring the tortuosity factor
- Brief review of experimental vs. theoretical status of slow wave
- Summary of the main themes
- Conclusions

Biot's (1962) Strain Energy Functional

$$2E = He^2 - 2Ce\zeta + M\zeta^2 - 4\mu I_2$$

where H, C, M, and μ are poroelastic constants,

$\varphi =$ porosity,

$\vec{u} =$ solid frame displacement,

$\vec{u}_f =$ pore fluid displacement,

$\vec{w} = \varphi(\vec{u}_f - \vec{u}) =$ relative displacement, and

$e = \nabla \cdot \vec{u} =$ frame dilatation,

$\zeta = -\nabla \cdot \vec{w} =$ increment of fluid content,

$I_2 = e_x e_y + e_y e_z + e_z e_x - \frac{1}{4}(\gamma_x^2 + \ldots) =$ a strain invariant.

Biot's Equations of Dynamic Poroelasticity

After first Fourier transforming from time to frequency:

$$\omega^2 \rho \vec{u} + (H - \mu)\nabla e + \mu \nabla^2 \vec{u} = -\omega^2 \rho_f \vec{w} + C\nabla\zeta,$$

$$\omega^2 q(\omega)\vec{w} - M\nabla\zeta = -\omega^2 \rho_f \vec{u} - C\nabla e,$$

where

$\omega = 2\pi f =$ angular frequency,

$\rho = \varphi\rho_{fluid} + (1-\varphi)\rho_{solid} =$ the average density,

$q(\omega) = \rho_f[\alpha/\varphi + iF(\xi)\eta/\kappa\omega]$, and

$\alpha =$ tortuosity, $\kappa =$ permeability, $F(\xi)\eta =$ dynamic viscosity,

$p_f = -M\nabla\cdot\vec{w} - C\nabla\cdot\vec{u} = M\zeta - Ce =$ fluid pressure.

Dispersion Relations for the Wave Solutions

- For shear wave (only one solution):

$$k_s^2 = \omega^2(\rho - \rho_f^2/q)/\mu$$

- For fast and slow compressional waves (quadratic: 2 solutions):

$$k_\pm^2 = \frac{1}{2}\left[b + f \mp [(b-f)^2 + 4cd]^{1/2}\right]$$

$$b = \omega^2(\rho M - \rho_f C)/\Delta, \quad c = \omega^2(\rho_f M - qC)/\Delta$$

$$d = \omega^2(\rho_f H - \rho C)/\Delta, \quad f = \omega^2(qH - \rho_f C)/\Delta$$

where

$$\Delta = HM - C^2.$$

Why is the Slow Wave Important?

• Demonstrated existence of the slow wave is one major test showing the validity of the equations of poromechanics.

• The direct connection of the slow wave to fluid flow in porous media suggests a number of ways it can be used to measure fluid permeability (Darcy's constant).

• Analysis of energy losses for seismic waves propagating through the earth requires us to account for all energy loss mechanisms, or false conclusions about significance of observations may occur.

• Mode conversions at all interfaces are important and must be accounted for properly. Many interfaces are present in the earth!

Slow Waves and Interfaces

J. Geertsma and D. C. Smit, "Some aspects of elastic wave propagation in fluid-saturated porous solids," *Geophysics* **26**, 169–181 (1961).

Quote: "Though in porous rocks the wave of the second kind can hardly be detected at some distance from the wave source, its existence cannot be entirely overlooked, as it must lead to absorption of a part of the total input energy. . . . a wave of the second kind is not only generated at a wave source, but also at any interface between dissimilar fluid-containing rocks. . . . in order to satisfy the boundary conditions at the interface" reflected and refracted waves of both kinds must be considered.

Structure Factor and Tortuosity (1)

One of the key parameters that determines the behavior of the slow wave in Biot's equations is the so-called "structure factor," later to be called the "tortuosity" or more precisely the "electrical tortuosity." Prior to the year 1980 when Plona published his results showing that a second compressional wave was observed in porous glass disks immersed in water, we had no methods of estimating this quantity α. But by the end of the year we had four physically distinct methods of estimating it.

Structure Factor and Tortuosity (2)

The four methods of estimating structure factor or tortuosity α are:

- Slow wave arrival times (via Plona's measurements)
- Electrical measurements of the electrical tortuosity (Brown)
- Liquid He II measurements of fourth sound at very low temperatures (Johnson)
- Effective density estimates from effective medium theory (Berryman)

Tortuosity from Slow Wave Speed (1)

At high (ultrasonic) frequencies ($\simeq$ 500 kHz and above), Biot's theory shows that the second compressional wave should have a wave speed given by

$$v_{eff} \simeq v_f/\sqrt{\alpha}.$$

So, by knowing the fluid wave speed v_f and measuring the effective wave speed of the slow compressional wave v_{eff}, we can infer the the value of the structure factor or electrical tortuosity α.

Tortuosity from Slow Wave Speed (2)

This approach follows easily from the diagram since the EIKONAL EQUATION is valid at high frequencies, and the slow wave is only weakly coupled to the surrounding frame material in this regime.

So, if T is the time of flight and v_f the fluid wave speed, we have

$$|\nabla T|^2 = 1/v_f^2.$$

But locally the eikonal equation is just given by

$$(T/L_e)^2 = 1/v_f^2,$$

while the apparent velocity v_e is the one actually measured at the macroscale and is given instead by

$$(T/L)^2 = 1/v_e^2.$$

Tortuosity from Slow Wave Speed (3)

So we have

$$v_e = v_f \times (L/L_e) = v_f/\tau_{sw},$$

where tortuosity (or index of refraction)

$$\tau_{sw} = L_e/L = 1/\cos\theta$$

as shown in the diagram. This calculation is valid for propagation through any one connected pore, but the slow wave is really a collective effect and so the slow-wave tortuosity is given more precisely by the (ensemble or path) averages

$$\tau_{sw} = \langle 1/\cos\theta \rangle = \left(\int ds/\cos\theta \right) / \int ds.$$

Finally, note that $\alpha = \tau_{sw}^2$, since $v_e = v_f/\sqrt{\alpha}$.

Tortuosity from Electrical Measurements (1)

The second, and in my opinion most unexpected, method of determining the tortuosity was first proposed by Brown in a paper in *Geophysics* in August, 1980. Because rocks, and many other porous media of interest such as sintered glass beads, are very good insulators, an electrical measurement of porous media — when saturated with a conducting fluid — is a measurement of a quantity called the "formation factor:"

$$F \equiv \sigma_0/\sigma_e,$$

where σ_0 is the conductivity of the pore fluid, and σ_e is the overall effective conductivity of porous rock saturated with pore fluid.

Tortuosity from Electrical Measurements (2)

Formation factor is a very well-known quantity in geophysics, as it is relatively easy to measure. For any value of porosity φ, the formation factor always satisfies

$$F \equiv \sigma_0/\sigma_e \geq 1.$$

Tortuosity from Electrical Measurements (3)

The power dissipated in an electrical system such as the one we are considering is determined by

$$P = \int d^3x J \cdot E,$$

where the electric field E is related to current distribution J by Ohm's law locally

$$J = \sigma_0 E,$$

and the electric field is related to the changes in local electrical potential Φ by

$$E = -\nabla\Phi.$$

Tortuosity from Electrical Measurements (4)

The power P is also related to the macroscopic current I through the electrodes and the potential difference $\Phi_1 - \Phi_2$ between the electrodes:

$$P = (\Phi_1 - \Phi_2)I.$$

The current I is related to the normal current density J by

$$I = JA,$$

where A is the area of the exit electrode.

Tortuosity from Electrical Measurements (5)

Now, using an argument very similar to the one given for the slow wave time of flight, we find that the current density inside the pores (see the diagram again) is given by

$$J = \sigma_0(\Phi_1 - \Phi_2)/L_e.$$

But, because the current only flows through the porosity (not the insulating solid frame), total exiting current is

$$I = J(\varphi A),$$

and so the total power actually dissipated in the whole volume is

$$P = \sigma_0[(\Phi_1 - \Phi_2)/L_e]^2 \varphi A \times L.$$

Tortuosity from Electrical Measurements (6)

But this power dissipation is interpreted macroscopically as if it were from a block of conducting material having σ_e conductivity and dimensions $L \times A$, so

$$P = \sigma_e[(\Phi_1 - \Phi_2)/L]^2 A \times L.$$

Setting these two expressions for power P equal, we have

$$\sigma_e = \sigma_0(L/L_e)^2\varphi.$$

The electrical tortuosity is then defined as

$$\alpha = (L_e/L)^2 = \varphi\sigma_0/\sigma_e = \varphi F,$$

where F is the formation factor.

Tortuosity and Liquid He II (1)

The third method of estimating/measuring tortuosity makes use of fourth sound in liquid He II at very low temperatures ($T < 1.1K$, near absolute zero).

Why fourth sound? What is fourth sound?

- First sound = pressure waves (normal and superfluid move in phase)
- Second sound = temperature or entropy waves (normal and superfluid oscillate out of phase, so no net transfer of matter)
- Third sound = surface wave on a liquid He film

Tortuosity and Liquid He II (2)

• Fourth sound = sound wave in a narrow channel where the normal component is stationary (due to viscous forces). Density and temperature both oscillate with the superfluid: $n = \sqrt{\alpha}$.

David Linton Johnson and colleagues have exploited fourth sound as a means of measuring the tortuosity. In a straight channel, the tortuosity α and the index of refraction are both equal to unity:

$$n = \sqrt{\alpha} = 1.$$

So the fourth sound speed v_4 is related only to the superfluid properties.

Tortuosity and Liquid He II (3)

But when the experiments are done in a superleak, or any typical porous medium like a glass-bead pack, the tortuosity is not unity and

$$v_e = v_4/n = v_4/\sqrt{\alpha}.$$

This method is therefore just like the first one we discussed, but uses a very exotic fluid in the pores. It has the advantage however that the fluid is nonviscous, so arrival amplitudes will be stronger than for any choice of viscous fluid.

Tortuosity from Solid Frame Inertia (1)

The fourth, and last, method of estimating the tortuosity is a theoretical method based on effective medium theory, and making use of still another physical point of view. Because there are two types of constituents in Biot's equations, there are three distinct types of inertia factors: ρ_{11}, ρ_{22}, and ρ_{12}, coupling solid-to-solid, fluid-to-fluid, and solid-to-fluid. There are two combinations of these that have simple interpretations:

$$\rho_1 = \rho_{11} + \rho_{12} = (1 - \varphi)\rho_{solid},$$

$$\rho_2 = \rho_{22} + \rho_{12} = \varphi\rho_{fluid},$$

the volume average inertias of the solid and fluid, respectively.

Tortuosity from Solid Frame Inertia (2)

Biot describes the quantity $-\rho_{12}$ as being the additional apparent density due to solid moving in the saturating (or immersing) fluid. Now it is well-known in fluid dynamics that solid objects oscillating in a fluid have a greater effective mass than they do when they are not immersed in the fluid. This effect is strongly shape dependent as can be seen by considering a solid disk: When it oscillates in the plane of the disk, it does not have to entrain very much fluid, but when it oscillates perpendicular to this plane it must push a great amount of fluid out of the way in order to move at all. So the effective mass of nonspherical objects is actually anisotropic.

Tortuosity from Solid Frame Inertia (3)

If we assume that the porous medium is itself isotropic, then we can also assume that the average mass induced by the surrounding fluid on the solid is also isotropic. In the simplest case, if the entire solid matrix is composed of spherical grains (consider a glass bead packing before sintering), then it should be a good approximation to suppose that the increased effective mass overall is comparable to that of each grain by itself. For spherical oscillators, the result is that effective density is density of the solid ρ_{solid} plus one-half of the density of the displaced fluid:

$$\rho_{11} = (1 - \varphi)(\rho_{solid} + \rho_{fluid}/2).$$

Tortuosity from Solid Frame Inertia (4)

The structure factor α is related to ρ_{12} by

$$\rho_{12} = -(\alpha - 1)\varphi\rho_{fluid}.$$

Putting all this information together, we can deduce that

$$\alpha = (1 + 1/\varphi)/2,$$

for granular media of the type considered. This result has proven to be of great practical use when direct measurements of α were not available.

Tortuosity from Solid Frame Inertia (5)

The formula

$$\alpha = (1 + 1/\varphi)/2,$$

has the very important feature that, as $\varphi \rightarrow 0$, the tortuosity goes to infinity. This feature is fundamentally correct, as the slow wave speed effectively goes to zero as the tortuosity goes to infinity, and porosity goes to zero.

There are simple ways to estimate α for granular media composed of nonspherical particles as well.

Frenkel and the Slow Wave (1)

J. Frenkel, "On the theory of seismic and seismoelectric phenomena in a moist soil," *J. Phys.* **8**, 230–241 (1944).

This paper was intended to explain observations by A. G. Ivanov in 1939 showing that "elastic waves in the surface layers of the soil" are accompanied by "electric potential differences between points situated at different distances from the source . . . ."

Frenkel and the Slow Wave (2)

Frenkel formulated a theory to account for these effects, and in particular found a quadratic equation that described the longitudinal waves in his model. This quadratic equation has two solutions:

"... one of them corresponds to waves with a very small damping, and the other – to waves with a very large damping. The waves of the second kind are thus really non-existant."

An Analogy from History of Astronomy (1)

Galileo Galilei (1564–1642) is both the first person to see the four largest moons of Jupiter through a telescope, and also to recognize that they were moons (not stars). So these moons are called the Galilean moons of Jupiter.

Simon Marius (1573–1624) independently discovered these moons of Jupiter the same year (1610). These moons are not named after him. But a long standing convention in astronomy is to permit the discoverer to suggest names for the objects discovered. So the names Io, Europa, Ganymede, and Callisto are in fact the ones suggested by Marius.

An Analogy from History of Astronomy (2)

Galileo also first saw the rings of Saturn. But his telescope did not have sufficient resolution to determine what the rings really were, and so he called them “the ears of Saturn” or “cup handles” because of what he saw through his telescope.

In contrast, Christiaan Huygens determined that these objects were rings around Saturn in 1655. Then, in 1675, Giovanni Domenico Cassini was able to resolve the rings sufficiently well to note that there were at least two rings. The largest space between rings is now called the “Cassini Division.” And so Galileo is usually not credited with the discovery of the rings.

MAIN THEMES

- Porous disks immersed in water bath
- Various methods of estimating the tortuosity
- Liquid He II measurements to reduce effects of viscosity
- Shock tube measurements
- Clay in rocks inhibits slow wave propagation
- Slow waves in air-filled rocks
- Attenuation losses attributed to slow waves at interfaces
- Two distinct slow waves in double-porosity media
- Electroseismic, seismoelectric lab and field experiments

POROMECHANICS:
SLOW WAVE REFERENCES (1)

1980 (Feb. 15) – Plona (slow wave first observed! but no theory!!)

1980 (Aug. 15) – Berryman (effective mass and "structure factor")

1980 (Dec. 15) – Johnson (liquid He, 4th sound, & slow wave) **

1981 – Salin and Schön (expt. confirmation of Plona's results)

1981 – Hovem (reflection and transmission coefficients)

1981 – Chandler (slow wave at low frequencies)

1981 – Chandler and Johnson (slow wave at low frequencies)

1982 – Johnson, Plona, Scala, Pasierb, & Kojima (liquid He II) **

1985 – Chin, Berryman, and Hedstrom (wave forms verified)

1985 – Van der Grinten, Van Dongen, & Van der Kogel (sh. tube)

1988 – Rasolofosaon (boundary condition tests)

1988 – Klimentos and McCann (clay vs. slow wave propagation)

POROMECHANICS: SLOW WAVE REFERENCES (2)

1990 – Nagy, Adler, and Bonner (slow waves in air-filled rocks)

1994 – Johnson, Plona, and Kojima (Plona's wave forms verified)

1995 – Gurevich and Lopatnikov (slow waves in layered media)

1996 – Nagy and Johnson (slow waves in air-filled media)

1997 – Kelder and Smeulders (slow waves in water-filled rocks)

1997 – Lafarge, Lemarinier, Allard, & Tarnow (air-filled media)

1999 – Shapiro and Müller (seismic attenuation and slow waves)

2000 – Berryman and Wang (slow waves in double-porosity media)

2002 – Pride, Tromeur, and Berryman (layered media)

2004 – Block (electrokinetics in granular media)

2004 – Haines (electroseismic field experiments)

2005 – Smeulders (slow wave review article)

Helium II:
Fourth Sound and the Slow Wave

1941 – Landau (quadratic equation for 1st and 2nd sound speeds)

1948 – Pellam (theory of mode conversion, 1st and 2nd sound)

1959 – Atkins (predicts surface and channel waves in He II) **

1962 – Rudnick & Shapiro (4th sound seen in packed powders) **

1965 – Shapiro & Rudnick (refraction index $n = (2 - \phi)^{1/2}$) **

1965 – Khalatnikov (textbook)

1968 – Fraser & Rudnick (porous glass vs. packed powders)

1980 – Johnson (liquid He, 4th sound, and the slow wave) **

1982 – Johnson, Plona, Scala, Pasierb, & Kojima (liquid He II) **

Fourth Sound Versus Slow Wave Observations

- Fourth sound was observed under several different circumstances in the 1960's (prior to Plona's measurements in 1980).
- Connection between these measurements and Biot's equations was apparently first made by David L. Johnson (1980).
- There are some differences in that liquid He is a mixed (viscous and nonviscous) fluid except at absolute 0K. Also, the analysis had not taken frame motion and tortuosity into account in a very significant way until Johnson's work. But very high porosity "index of refaction" had been measured for superleaks.

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CONCLUSIONS

- Since 1980, the slow wave has been observed many times in many different circumstances by many people.
- Slow wave behavior is especially subtle in rocks because there are various sources of contamination that prevent the simplest version of the theory from being directly applicable. There are very specific contaminates such as clays that have been shown to inhibit slow wave propagation. Other issues concerning up-scaling strongly suggest that the simplest versions of the equations are probably not adequate at large scales.
- New field methods, such as seismoelectric approach, are continuing to be developed and offer some new possibilities for avoiding some of the past difficulties with field measurements.

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Helium II:
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