Poromechanical processes in fault zones during earthquake rupture

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A quick primer on the structure of mature, highly slipped faults:

Wide damage zones but extreme localization of seismic slip
Fig. 1. Generalized geologic map of the San Gabriel Mountains and vicinity, southern California. Stippled pattern represents crystalline rocks. Bold arrows indicate the study localities discussed in the text: Devil's Canyon (DC), Bear Creek (BC), and Punchbowl (P). Key: active trace of the San Andreas fault (SAF), San Gabriel fault (SGF), North Branch San Gabriel fault (NBGF), South Branch San Gabriel fault (SBSGF), Punchbowl fault (PF), Sierra Madre-Cucamonga thrust (SMCT), San Antonio fault (SA), Vincent thrust (VT), Fenner fault (FF), Soledad fault (SF), San Francisquito fault (SFF), Ridge basin (RB), Soledad basin (SB) and Punchbowl basin (PB).
Generic structure, mature fault zones:


Internal Structure of Principal Faults of the North Branch San Gabriel Fault

Fig. 2. Schematic section across the North Branch San Gabriel fault zone illustrating position of the structural zones of the fault. The diagram is not to scale.

- ~10-100 m (Damage ≈ highly cracked rock.)
- 1-10 m (Granulated, fault gouge)
- 10s-100s mm (But principal failure surface can be much thinner, typically < 1-5 mm!)
Extreme localization of slip (several km) on mature fault -- Punchbowl Fault
[Chester & Chester, 1998; Chester et al., 2003, 2004; Chester and Goldsby, 2003]

Figure 1: Principal slip surface (pss) along the Punchbowl fault. (a) From Chester and Chester [1998]: Ultracataclasite zone with pss marked by black arrows; note 100 mm scale bar. (b) From Chester et al. [2005a] (also, Chester et al. [2003] and Chester and Goldsby [2003]): Thin section; note 5 mm scale bar and ~1 mm localization zone (bright strip when viewed in crossed polarizers due to preferred orientation), with microshear localization of most intense straining to ~100-300 μm thickness.
Particle size distribution for ultracataclasite hosting the Punchbowl pss
[J. Chester et al., *Nature*, 2005]

- $N(D)/A = \text{number of particles per unit sample area with } 2D/3 < \text{diameter} < 4D/3$.

- $N(D)/A \approx c/D^2$ for $\sim 10 - 30 \text{ nm} < D < \sim 70 \mu\text{m}$.

- $D_{50} (= \text{size such that 50% by wt. is larger}) \sim 1 \mu\text{m}$.

- *If* standard granular material guidelines hold, thinnest possible shear zone should be $\sim 10D_{50} \approx 10 \mu\text{m}$.

(But $D = 10 \mu\text{m}$ corresponds to $D_{20}$ to $D_{25}$! Clumping of small cohesive-sized particles likely.)
Quandary in seismology:

Lab estimates of rock friction coefficient $f$ usually high, $f \sim 0.6-0.8$.

Fault slip zones are thin.

$\Rightarrow$ If those $f$ prevail during seismic slip, we should find

- measurable heat outflow near major faults, and

- extensive melting along exhumed faults.

Neither effect is generally found.
One line of explanation: Weak faults:

- Fault zone materials are different, have very low $f$, e.g., $\sim 0.1-0.2$.
- $f$ isn’t low, but pore pressure is high over much of the fault.

Another line: Sastically strong but dynamically weak faults:

- Thermal weakening in rapid, large slip:
  - Flash heating of asperity contacts, reduces $f$ in rapid slip.
  - Thermal pressurization of pore fluid, reduces effective stress.
  - Thermal decomposition, fluid product phase at high pressure.
  - Gel(?) formation at large slip in wet silica-rich faults.
  - Melting at large slip, if above set has not limited increase of $T$.

- Across-fault elastic dissimilarity of adjoining crustal blocks and/or poroelastic/permeability dissimilarity of saturated damage fringes: Both $\Rightarrow$ mode II slip can reduce $\sigma_n - p$. 
Flash heating of microscopic frictional asperity contacts

\[ V = \text{slip rate} \]

\[ \dot{\sigma} \text{ macro-stresses} \]

\[ \tau \text{ contact-stresses} \]

\[ D \text{ asperity diameter} \]

\[ T = \text{asperity temperature} \]

\[ T_f = \text{average temperature of fault surfaces} \]

\[ \tau_c \text{ shear stress} \]

\[ T, \text{ asperity temperature} \]

\[ T_f, T_w, \text{ weakening temperature} \]
We(7,7),(993,986)(8,3),(989,990)akening velocity $V_w$ : When $V > V_w$, asperity of size $D$ weakens ($T \rightarrow T_w$, and contact strength $\rightarrow$ small value) before end of contact lifetime $D / V$:

$$V_w = \frac{\pi \alpha_{th}}{D} \left[ \frac{\rho c(T_w - T_f)}{\tau_c} \right]^2$$

**Example :**

- $\alpha_{th} = 0.5$ mm$^2$/s, $\rho c = 2.7$ MJ/m$^3$K,
- $D = 5$ µm, $T_w = 900$ºC,
- and $\tau_c = 0.1 \times$ (elastic shear modulus) = 3.0 GPa

$\Rightarrow V_w \approx 0.20$ m/s at $T_f = 20$ºC

$(\Rightarrow V_w \approx 0.12$ m/s at $T_f = 200$ºC)

Fits to Tullis & Goldsby [2003] room temperature data (for quartzite, granite, feldspar, gabbro, calcite):

$$V_w \approx (0.14, 0.14, 0.28, 0.11, 0.27) \text{ m/s}$$

(Rice [EOS, 1999; JGR, 2006], 1-D heat flow analysis like by Archard [Wear, 1958-59])
Recent lab set-ups to study weakening at high slip rates

Rotary Shear Apparatus

High speed $V \leq 0.36 \text{ m/s}$

$\sigma_n = 5 \text{ MPa}$

**Instron testing frame, rotary shear of annular ring up to 45 mm slip**


Results for **Arkansas novaculite (quartzite)**, granite, **Tanco albite (feldspar)**, gabbro, and calcite.

Slip rate $V$ up to $\sim 4 \text{ m/s}$, $\sigma_n$ up to 10’s of MPa

**Torsional Kolsky bar apparatus**


Results for **Arkansas novaculite (quartzite)** only.
Weakening of friction coefficient \( f \) at high slip rates

Results here for *Arkansas novaculite* (~100% quartzite), determined in rotating annular specimens

Tullis and Goldsby (2003a,b), up to 0.36 m/s

Prakash (2004), 2 to 4 m/s

Slip rates \( V \) up to 0.36 m/s imposed in Instron testing frame for 45 mm slip, after 1.2 mm pre-slip at \( \sim 10 \mu \text{m/s} \). At low \( V \), friction coefficient \( f \approx 0.65 \), whereas at \( V > 0.3 \text{ m/s} \), \( f \approx 0.3 \).

Comparable rate-weakening was found for *granite*, *Tanco albite* (~100% feldspar), and *gabbro*, but ambiguous results for *calcite*.

Pre-twisted torsional Kolsky bar (split Hopkinson bar) imposes slip at \( V \sim 3-4 \text{ m/s} \), resulting in \( f \) slightly less than 0.2.

Experiment becomes uninterpretable after small slip (marked) due to cracking in wall of specimen.
**Model** for $f_{ss}$ (*steady state* friction)

[Beeler and Tullis, 2003, 2006; Rice, 1999, 2006]:

$$f_{ss} = \begin{cases} f_o, & V \leq V_w, \\ f_w + (f_o - f_w) \frac{V_w}{V}, & V \geq V_w, \end{cases}$$  with  $V_w = \pi \frac{\alpha_{th} (T_w - T)}{D \left( \frac{T_c}{\rho c} \right)^2}$

*Fits to Tullis & Goldsby [2003] room temperature data, $V = 0$ to 0.36 m/s*

*For quartzite, granite, feldspar, gabbro, calcite:*

$V_w \approx (0.14, 0.14, 0.28, 0.11, 0.27)$ m/s

*For quartzite, granite, gabbro:*

$f_o \approx (0.64, 0.82, 0.88), \; f_w \approx (0.12, 0.13, 0.15)$
Thermal pressurization of pore fluid
Governing equations, 1-space-dimension shearing field, constant normal stress $\sigma_n$:

\[
\begin{align*}
\tau & = \eta \left( \frac{\partial u}{\partial y} \right) \\
\sigma_n & = -\sigma_{yy} \\
\end{align*}
\]

**Energy equation**: 
\[
\tau \dot{\gamma} = \rho c \frac{\partial T}{\partial t} + \frac{\partial q_h}{\partial y}, \quad q_h = -K \frac{\partial T}{\partial y},
\]
\[
\dot{\gamma} = \frac{\partial u}{\partial y} \geq 0, \quad \tau = f(\sigma_n - p) \text{ when } \dot{\gamma} > 0:
\]
\[
f(\sigma_n - p)\dot{\gamma} = \rho c \frac{\partial T}{\partial t} - \frac{\partial}{\partial y} \left( \rho c \alpha_{th} \frac{\partial T}{\partial y} \right);
\]
\[
\rho c \approx 2.7 \text{ MPa/}^\circ\text{C}; \quad \alpha_{th} = \frac{K}{\rho c} \approx 0.5-0.7 \text{ mm}^2/\text{s}.
\]

**Fluid mass conservation**: 
\[
\frac{\partial m}{\partial t} + \frac{\partial q_f}{\partial y} = 0, \quad q_f = -\frac{\rho_f k}{\eta_f} \frac{\partial p}{\partial y} \Rightarrow
\]
\[
\frac{\partial p}{\partial t} = \Lambda \frac{\partial T}{\partial t} - \frac{1}{\beta} \frac{\partial \rho_f}{\partial t} + \frac{1}{\rho_f \beta} \frac{\partial}{\partial y} \left( \rho_f \beta \alpha_{hy} \frac{\partial p}{\partial y} \right),
\]
\[
\alpha_{hy} = k / \eta_f \beta;
\]
\[
\Lambda \approx 0.3-1.0 \text{ (MPa/}^\circ\text{C)}, \quad \beta \equiv n(\beta_f + \beta_n) \approx 5-30 \times 10^{-11} /\text{Pa};
\]
\[
\beta_f, \beta_n = \text{fluid compressibility, pore space expansivity.}
\]
**Slip on a plane** at slip rate $V$ \( ( \text{Thickness } h \text{ of shearing layer assumed small compared to boundary layers where } p \text{ and } T \text{ increase} ) \):

- In \( |y| > 0 \), \( \frac{T}{t} = \alpha_{th} \frac{2T}{y^2} \) and \( \frac{p}{t} - \Lambda \frac{T}{t} = \alpha_{hy} \frac{2p}{y^2} \).

- On \( y = 0^\pm \), \( q_h = -K \frac{T}{y} = \frac{f(n - p)V}{2} \); \( q_f = -\frac{\rho_f}{\eta_f} \frac{p}{y} = 0 \).

- Assumes all dilatancy $\Delta n^{pl}$ (distributed) is over at small slip \( [p_{amb} \rightarrow p_o = p_{amb} - \frac{\Delta n^{pl}}{\beta}] \).

**Simple solution:** For \( V \equiv d\delta/ dt = \text{constant} \), and \( f = \text{constant} \), we solve for the fields \( T(y,t) \) and \( p(y,t) \), and hence \( p(0,t) \), to evaluate

\[
\tau = \tau(\delta) = f(\sigma_n - p(0,t)) \quad \text{(where slip } \delta = Vt) : \\
\tau(\delta) = f(\sigma_n - p_o) \exp\left(\frac{\delta}{L^*}\right) \text{erfc}\left(\sqrt{\frac{\delta}{L^*}}\right),
\]

where \( L^* = \frac{4\left(\rho c / \Lambda\right)^2}{f^2\left(\sqrt{\alpha_{hy}} + \sqrt{\alpha_{th}}\right)^2 \frac{V}{V}} \).

[Rice, *JGR*, 2006, generalizing solution with \( \alpha_{hy} = 0 \) by Mase & Smith, *JGR*, 1987]
Simple solution: Slip on a Plane, $\dot{\gamma} = V \delta_{\text{Dirac}}(y)$, stress vs. slip (for $V = \text{constant} & f = \text{constant}$):

$$\frac{\tau}{f(\sigma_n - p_o)} = \exp\left(\frac{\delta}{L^*}\right) \text{erfc}\left(\frac{\sqrt{\delta}}{L^*}\right); \quad L^* = \frac{4}{f^2} \left(\frac{\rho c}{\Lambda}\right)^2 \left(\sqrt{\alpha_{hy}} + \sqrt{\alpha_{th}}\right)^2.$$

(Assumes all dilatancy $\Delta n^{pl}$ (distributed) is over at small slip [$p_{amb} \rightarrow p_o = p_{amb} - \frac{\Delta n^{pl}}{\beta}$].)

Note apparent multi-scale nature of the slip-weakening; no well-defined $D_c$:
How large is $L^*$?

$$L^* = \frac{4}{f^2} \left( \frac{\rho c}{\Lambda} \right)^2 \left( \sqrt{\alpha_{hy}} + \sqrt{\alpha_{th}} \right)^2. $$

Evaluations for 7 km depth, a typical centroidal depth of crustal rupture zone; 
$\sigma_n \approx$ overburden = 196 MPa, $p_o = p_{amb} =$ hydrostatic = 70 MPa, $T_{amb} \approx 210 \degree C$:

• Part of $L^*$ based on poro-thermo-elastic properties of fault gouge:

Assuming relatively intact gouge, MTL lab-measured properties [Wibberley & Shimamoto, JSG, 2003]. 

$$\left( \frac{\rho c}{\Lambda} \right)^2 \left( \sqrt{\alpha_{hy}} + \sqrt{\alpha_{th}} \right)^2 \approx \begin{cases} 
60 \text{ mm}^2 /s \text{ (low end)} \Rightarrow L^* \approx 4 \text{ mm}, \text{ if } V = 1 \text{ m/s and } f = 0.25. \\
450 \text{ mm}^2 /s \text{ (high end)} \Rightarrow L^* \approx 30 \text{ mm}, \text{ if } V = 1 \text{ m/s and } f = 0.25.
\end{cases}$$

Accounting arbitrarily for damage at the rupture front and during subsequent shear, permeability $k_{dmg} = 5-10 \times k_{intact}$, 
drained skeletal compressibility $\beta_{d_{dmg}} = 1.5-2 \times \beta_{d_{intact}}$.

• $V \approx 1 \text{ m/s}$ (For the 7 slip inversions [Heaton, 1990]: ratio of slip distance to slip duration at a point is 0.56 to 1.75 m/s, average is 1.06 m/s). 
[Slip durations could be unresolvably shorter, so $V \approx 1 \text{ m/s}$ may be lower bound.]

• $f \approx 0.25$ represents effects of flash heating, like in high-speed friction experiments [Tullis and Goldsby, 2003; Prakash, 2004].
Comparison of predictions to seismic constraints on fracture energy $G$

From theoretical modeling, we can predict $\tau = \tau(\delta)$ for slip $\delta$ at a constant rate $V$, representative of that in earthquakes ($\delta = Vt$).

Corresponding slip-weakening fracture energy $G = G(\delta)$ then predicted from

$$ G = G(\delta) \equiv \int_0^\delta [\tau(\delta') - \tau(\delta)]d\delta' \quad (\delta = \text{total slip}). $$

[This ignores contributions to $G$ from dissipation in off-fault plastic zones; important to understand how much!]
Comparison, predictions of fracture energy \( G = G(\delta) \equiv \int_{0}^{\delta} [\tau(\delta') - \tau(\delta)] d\delta' \) (\( \delta = \) slip) from the slip-on-a-plane model with seismic estimates, for:

• A large-earthquake data set [Rice, Sammis & Parsons, BSSA, 2005; Tinti, Spudich, & Cocco, JGR, 2005] for \( G \) from seismic slip inversions for 12 events (shown as ovals).

• A data set for \( G' \) for small and large events based on radiated energy, moment, and seismic source dimension [Abercrombie & Rice, GJI, 2005]; \( G' \approx G \), and \( G' = G \) if stress during last increments of slip = final static stress after rupture (no overshoot/undershoot).
A perspective on extreme localization of seismic slip in fluid-infiltrated fault gouge

[work in progress, Rice, Rudnicki & Tsai]
Governing equations, 1-space-dimension shearing field, constant normal stress $\sigma_n$:

**Thermal pressurization of pore fluid**

(Habib 67, 75, Sibson 73, Anderson 80, Lachenbruch 80, Voight & Faust 82, Mase & Smith 85, 87, Lee & Delaney 87, Vardoulakis 02, Andrews 04, many more in recent yrs)

- Energy equation:
  \[
  \tau \dot{\gamma} = \rho_c \frac{\partial T}{\partial t} + \frac{\partial q_h}{\partial y}, \quad q_h = -K \frac{\partial T}{\partial y},
  \]
  \[
  \dot{\gamma} = \frac{\partial u}{\partial y} \geq 0, \quad \tau = f(\sigma_n - p) \text{ when } \dot{\gamma} > 0:
  \]
  \[
  f(\sigma_n - p)\dot{\gamma} = \rho_c \frac{\partial T}{\partial t} - \frac{\partial}{\partial y} \left( \rho_c \alpha_{th} \frac{\partial T}{\partial y} \right);
  \]
  \[
  \rho_c \approx 2.7 \text{ MPa/°C}; \quad \alpha_{th} = \frac{K}{\rho_c} \approx 0.5-0.7 \text{ mm}^2/\text{s}.
  \]

- Fluid mass conservation:
  \[
  \frac{\partial m}{\partial t} + \frac{\partial q_f}{\partial y} = 0, \quad q_f = -\frac{\rho_f k}{\eta_f} \frac{\partial p}{\partial y} \Rightarrow
  \]
  \[
  \frac{\partial p}{\partial t} = \Lambda \frac{\partial T}{\partial t} - \frac{1}{\beta} \frac{\partial n_{pl}}{\partial t} + \frac{1}{\rho_f \beta} \frac{\partial}{\partial y} \left( \rho_f \beta \alpha_{hy} \frac{\partial p}{\partial y} \right),
  \]
  \[
  \alpha_{hy} = k / \eta_f \beta;
  \]
  \[
  \Lambda \approx 0.3-1.0 \text{ (MPa/°C)}, \quad \beta \equiv n(\beta_f + \beta_n) \approx 5-30 \times 10^{-11} /\text{Pa};
  \]
  \[
  \beta_f, \beta_n = \text{fluid compressibility, pore space expansivity}.
  \]
The spatially uniform solution:
Adiabatic, undrained, uniform shear, at constant shear rate

\[ u(y,t) = u_o(y) = \dot{\gamma}_o y \quad (\dot{\gamma}_o = \text{uniform shearing rate}), \]

\[ p(y,t) = p_o(t) \quad , T(y,t) = T_o(t). \]

- \( \tau(y,t) = \tau_o(t) = f \left( n - p_o(t) \right) \) (we assume \( f = \text{constant in large shear with } \dot{\gamma} = \text{constant} \))
  
  \[ \frac{\tau_o(t)\dot{\gamma}_o}{\rho c} = \frac{dT_o(t)}{dt} \]

- \[ \frac{dp_o(t)}{dt} = \Lambda \frac{dT_o(t)}{dt} - \frac{1}{\beta} \frac{dn^{pl}}{dt} \] (we assume \( \frac{dn^{pl}}{dt} = 0 \) in large shear with \( \dot{\gamma} = \text{constant} \))

**Solution** (Lachenbruch [1980]):

\[ \sigma_n - p_o(t) = (\sigma_n - p_o(0)) \exp \left( -\frac{f \Lambda}{\rho c} \dot{\gamma}_o t \right) \quad [\text{call this } \bar{\sigma}_o(t)] \]

\[ \tau_o(t) = f \bar{\sigma}_o(t) \quad (\text{weakening strain } \frac{\rho c}{f \Lambda} \approx 10-20, f \approx 0.4) \]
Is the spatially homogeneous solution stable?

Answer is no. Shear should localize to a mathematical plane if \( f = \text{constant} \) and \( \partial n^{pl} / \partial t = 0 \).

Aim in our work [Rice, Rudnicki and Tsai, EOS, F05 AGU]:

Add (separately) two stabilizing features.

Does shear then localize? If so, what thickness?

Features are:

(1) **Rate-strengthening friction coefficient**, \( f = f(\dot{\gamma}) \) with \( df(\dot{\gamma}) / d\dot{\gamma} > 0 \).

   Approximately valid only in stable regions in which rupture cannot nucleate, but may propagate through, or in unstable regions that have shear-heated to a frictionally stable \( T \) range.

(2) **Dilatancy, increasing with rate of sustained shear**, \( n^{pl} = n^{pl}(\dot{\gamma}) \) with \( dn^{pl}(\dot{\gamma}) / d\dot{\gamma} > 0. \) (not further discussed here)

For simplicity, treatments ignore rate and state transients in response to changes in shearing rate, and use "steady state" response in defining \( f(\dot{\gamma}) \) and \( n^{pl}(\dot{\gamma}) \).
Linearized perturbation about time-dependent spatially uniform solution:

\[ u(y,t) = \dot{\gamma}_o y + u_1(y,t) , \]
\[ \dot{\gamma}(y,t) = \partial u(y,t) / \partial y = \dot{\gamma}_o + \dot{\gamma}_1(y,t) , \]
\[ p(y,t) = p_o(t) + p_1(y,t) , \]
\[ T(y,t) = T_o(t) + T_1(y,t) , \]
\[ f = f(\dot{\gamma}_o) + f'(\dot{\gamma}_o) \dot{\gamma}_1(y,t) \]

Nature of solution with spatial dependence \( \exp(2\pi iy/\lambda) \) (wavelength \( \lambda \)):

\[ p_1(y,t), T_1(y,t) \propto \exp(st) \exp(2\pi iy/\lambda) ; \]
\[ \dot{\gamma}_1(y,t) \propto \exp\left[ \left( s + \frac{fA}{\rho c} \dot{\gamma}_o \right) t \right] \exp(2\pi iy/\lambda) \]

\( s = s(\lambda) \) satisfies:

\[
z \frac{fA}{\rho c} \dot{\gamma}_o s = \left( s + \frac{4\pi^2 \alpha_{th}}{\lambda^2} \right) \left( s + \frac{4\pi^2 \alpha_{hy}}{\lambda^2} \right) \quad \text{where} \quad z = \frac{f(\dot{\gamma}_o)}{\dot{\gamma}_o f'(\dot{\gamma}_o)} = \frac{f}{a-b} \approx \frac{0.6}{0.015} = 40
\]
Condition for linear instability of flow profile:

$$\text{Re}(s(\lambda)) + \frac{f \Lambda}{\rho c} \dot{\gamma}_o > 0 \quad \Rightarrow \quad \lambda > \lambda_{cr} \equiv 2\pi \sqrt{\frac{\rho c}{f \Lambda} \frac{(\alpha_{th} + \alpha_{hy})}{(z + 2)\dot{\gamma}_o}} \quad \left( z = \frac{f(\dot{\gamma}_o)}{\dot{\gamma}_o f'(\dot{\gamma}_o)} \gg 1 \right)$$

Comment: Near $\lambda = \lambda_{cr}$, $\text{Im}(s(\lambda)) \approx z \frac{\sqrt{\alpha_{th} \alpha_{hy}}}{\alpha_{th} + \alpha_{hy}} \frac{f \Lambda}{\rho c} \dot{\gamma}_o$ (oscillatory; unloading?)

$$\begin{bmatrix} \text{fast; compare to } \sigma_o(t) \propto \exp\left(-\frac{f \Lambda}{\rho c} \dot{\gamma}_o t\right) \end{bmatrix}$$

Shear of a rate-strengthening layer of thickness $h$ at net "slip" rate $V$ ($\dot{\gamma}_o = V/h$):

$$\lambda > \lambda_{cr} \equiv 2\pi \sqrt{\frac{\rho c}{f \Lambda} \frac{(\alpha_{th} + \alpha_{hy})h}{(z + 2)V}}$$
Linear Stability Analysis [Rice & Rudnicki, in preparation]

Shear of a rate-strengthening layer of thickness $h$ at net "slip" rate $V$ ($\dot{\gamma}_o = V / h$).

Perturbation solution with spatial dependence $\exp(2\pi iy / \lambda)$ (wavelength $\lambda$):

Stable if $\lambda < \lambda_{cr}$, Unstable if $\lambda > \lambda_{cr}$.

\[
\lambda_{cr} = h \Rightarrow h = 4\pi^2 \frac{\rho c (\alpha_{th} + \alpha_{hy})}{f \Lambda (z + 2)V}
\]

\[
\alpha_{th} = 1 \text{ mm}^2/\text{s}
\]

\[
V = 1 \text{ m/s}
\]

\[
z = 40
\]

drawn for:
Nonlinear numerical analysis: Typical dynamics

[V. Tsai, studies in progress, 2005-] $T, p$, and $(\sigma_n - p)$ parameter dependence:
Blanpied et al. [1998], Keenan et al. [1978]; Wibberley & Shimamoto [2003]
Dynamic rupture simulations, using lab-constrained poroelastic, thermal, and friction parameters

(combining flash heating and thermal pressurization of pore fluid)

[Noda, Dunham & Rice, EOS, 2006, and work in progress]

Examples to follow:
\( \delta = \delta(x,t), \ \tau = \tau(x,t); \)
slip and shear stress in \( z \) direction (mode III)
Elastodynamics

Relates history of slip $\delta(x,t)$ to history of stress $\tau(x,t)$ [recall, $\tau = f(\sigma_n - p)$] :

$$\tau(x,t) = \tau_o(x,t) - \frac{\mu}{2c_s} V(x,t) + \phi(x,t)$$

where

$$V(x,t) = \frac{\partial \delta(x,t)}{\partial t},$$

equals background stress $\tau^b$ outside nucleation zone

$$\phi(x,t) = \text{linear functional of slip history } \delta(x',t') \text{ within wave cone of } (x,t):$$

Could use BIE, FD, FE, or Spectral formulation;

Spectral (for slip between uniform half spaces) is :

$$\begin{bmatrix} \delta(x,t) \\ \phi(x,t) \end{bmatrix} = \sum_n \begin{bmatrix} D_n(t) \\ \Phi_n(t) \end{bmatrix} \exp(in\hat{k}x)$$

$$\Phi_n(t) = \int_0^t C_n(t-t') \dot{D}_n(t') dt'$$

Need a constitutive relation between $V(t)$ and $\tau(t)$ at each $x$ to close the system.
**Flash heating**

Effective stress law:
\[
\tau = \sigma_e f = (\sigma_n - p|_{z=0}) f
\]

Flash heating at microscopic contacts, model for \( f_{ss} \) (steady state friction)


\[
f_{ss} = \begin{cases} 
fo, & V \leq V_w, \\
fw + (fo - fw) \frac{V_w}{V}, & V \geq V_w,
\end{cases}
\]

with \( V_w = \pi \frac{\alpha_{th}}{D} \left( \frac{T_w - T}{\tau_c / \rho c} \right)^2 \)

Rate- and state-dependent friction law (in form of “slip law” [Dietrich, 1979; Ruina, 1983])

[Something like this needed to regularize problem; remove short wavelength blowup]
\[
\frac{df}{dt} = \frac{a}{V} \frac{dV}{dt} - \frac{V}{L} (f - f_{ss}) \quad \text{(with } L \sim 5-10 \ \mu\text{m})
\]
Thermal pressurization

Effective stress law:

\[ \tau = \sigma_e f = \left( \sigma_n - p \right)_{z=0} f \]

Thermal pressurization (T.P.), infinitesimally thin slipping zone
(later, we will see finite thickness cases)

Conservation of energy

\[ \frac{\partial T}{\partial t} = \alpha_{th} \frac{\partial^2 T}{\partial z^2} \]

\[ \alpha_{th} \rho c \frac{\partial T}{\partial z} \bigg|_{z=0} = -\frac{1}{2} f \sigma_e V \]

B.C.

Conservation of fluid mass

\[ \frac{\partial p}{\partial t} = \alpha_{hy} \frac{\partial^2 p}{\partial z^2} + \Lambda \frac{\partial T}{\partial t} \]

\[ \frac{\partial p}{\partial z} \bigg|_{z=0} = 0 \]
**Numerical implementation**

**Multiple time steps**

- Longer timesteps: history storage and convolution
- Shorter timesteps: state, T.P. integration

**Guess displacement, velocity field**

**Linear interpolation of stress transfer**

Integrate $\delta, \Psi, T, \tau$ and $p$ solving elasticity and constitutive law

**Better guess of stress transfer**

Obtain final value
Physical parameters

State evolution distance: $L = 5 \, \mu m$ unless otherwise noted.

Hydro–thermal diffusivity factor:

$$\left( \frac{\rho c}{\Lambda} \right)^2 \left( \sqrt{\alpha_{th}} + \sqrt{\alpha_{hy}} \right)^2 = 20 \sim 460 \, \text{mm}^2/\text{s}$$

a little wider range than earlier

by setting a **degree of damage**, $r \in [0, 1]$

$$\alpha_{hy} = 0.86 + 2.66 \, r \, \text{mm}^2/\text{s}$$

$$\Lambda = 0.98 - 0.64 \, r \, \text{MPa/K}$$

Other physical parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\rho c$</td>
<td>2.7 MJ/m$^3$K</td>
</tr>
<tr>
<td>$T_w$</td>
<td>900 °C</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>3 GPa</td>
</tr>
<tr>
<td>$c_s$</td>
<td>3 km/s</td>
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<tr>
<td>$f_o$</td>
<td>0.82</td>
</tr>
<tr>
<td>$f_w$</td>
<td>0.13</td>
</tr>
</tbody>
</table>

From granite flash weakening data, Beeler and Tullis, 2003
Effect of hydrothermal properties

Mode III, Different $\alpha_{th}$ and $\Lambda$

$\tau_{\text{per}} = 30\text{MPa}$
$D_{\text{per}} = 5\text{cm}$

Background shear stress, $\tau^b / \sigma_{e\theta}$

H.T. diffusivity factor, mm$^2$/s

Arrested pulse
Growing pulse
Crack-like

Below $\tau_{\text{pulse}}$

Damaged
Intact

Thermal pressurization favors crack-like solution.
Effect of hydrothermal properties on different $\alpha_{th}$ and $\Lambda$

$\tau_{per} = 30\text{MPa}$
$D_{per} = 5\text{cm}$

Arrested pulse
Growing pulse

Thermal stress, crack-like solution.
Effect of hydrothermal properties

Mode III, Different $\alpha_{th}$ and $\Lambda$

$\tau_{per} = 30$MPa
$D_{per} = 5$cm

Background shear stress, $\tau^b / \sigma_{e0}$

H.T. diffusivity factor, mm$^2$/s

Thermal pressurization favors crack-like solution.
Slip distribution, pore pressure rise

**Growing pulse**
- **Displacement increases.**
- **Slip every 20 μs**
  - $r = 0.74$

**Crack**
- **p continues to increase.**
  - $r = 0.72$

**Strength recovery ~ 100 μs**
- Slip rate
- Pore pressure at $z = 0$

**Time, μs**
- 300
- 400
- 500
- 600
- 700

**Pore pressure, MPa**
- 0
- 20
- 40
- 60
- 80
- 100

**Slip rate, m/s**
- 0
- 20
- 40
- 60
- 80
- 100
Natural earthquakes seem best described by the *self-healing* mode [Heaton, *EPSL*, 1990]

Example from set of seven large earthquakes studied, with similar (if generally less extreme) results, Heaton [*EPSL*, 1990]
Finite thickness of slipping zone

Thermal pressurization

\[ \frac{\partial T}{\partial t} = \alpha_{th} \frac{\partial^2 T}{\partial z^2} + \frac{\omega}{\rho c} \]

\[ \frac{\partial T}{\partial z} \bigg|_{z=0} = 0 \]

\[ \frac{\partial p}{\partial t} = \alpha_{hy} \frac{\partial^2 p}{\partial z^2} + \Lambda \frac{\partial T}{\partial t} \]

\[ \frac{\partial p}{\partial z} \bigg|_{z=0} = 0 \]

where \( \omega \) is heat source distribution:

\[ \omega = \begin{cases} 
  f(\sigma_n - p(0,t)) \frac{V}{2w} & (z < w) \\
  0 & (z > w) 
\end{cases} \]

\( w \): half width of slipping zone

We assume uniform heat generation within a slipping zone.
Boundary with damaged ($r = 1$) hydraulic properties

Slip distribution every 50 μs

$\frac{\tau_b}{\sigma_{e0}} = 0.2460$

$2w = 0 \mu m$

Slip, mm

Distance, m

Pore pressure rise at $x = 1$ m

$2w = 0 \mu m$

$2w = 50 \mu m$

$2w = 100 \mu m$

$2w = 200 \mu m$

Pore pressure rise, MPa

Time, μs

Mode III, Different $\alpha_{th}$ and $\Lambda$

Background shear stress, $\frac{\tau_b}{\sigma_{e0}}$
At the farthest point from the boundary, but still below $\tau_{\text{pulse}}$.

Thickness of slipping zone has a great effect!
Accounting for inelastic effects in fluid-saturated damage zones, Mohr-Coulomb elastic-plastic analysis of rupture propagation

[Templeton, Viesca & Rice, EOS, 2006, 2007 & in progress]
Analyses of right-lateral rupture in an elastic-plastic medium are conducted using ABAQUS/Explicit in the framework of 2-dimensional plane strain. Linear slip-weakening behavior along the fault is implemented with a split-node procedure. The mesh is refined so that there are 20 elements within the $R_0$, the size of the static slip-weakening zone. Rupture is nucleated along the fault by introducing an initial slip as in Kame et al. [JGR, 2003] within a region of length $L_c'$, which is slightly larger than $L_c$, causing an initial stress concentration, which is slightly larger than the peak strength, at both tips of the static nucleation zone.
Off-fault elastic-plastic material behavior

Pressure-Dependent Yield Surface

Shear strain increment:
\[ dy = \frac{1}{G} d\tau + \frac{1}{h} (d\tau - \mu d\sigma) \]

Volumetric strain increment:
\[ d\varepsilon = -\frac{1}{K} d\sigma + \frac{\beta}{h} (d\tau - \mu d\sigma) \]

Inelastic deformation of the off-fault material is described by a pressure-dependent yield criterion. The yield surface proposed by Drucker and Prager [Q. Appl. Math, 1952], a modification of the Huber-von Mises yield criterion, is a simple choice for describing granular materials such as the granulated material of damage zones along faults, which exhibit pressure-dependent yielding. Plastic deformation is perfectly plastic (no hardening) and can be dilatant ($\beta > 0$).

Drucker Prager Yield Criterion

Yield Function:
\[ F = \bar{\tau} + \mu \left( \frac{1}{3} \sigma_{kk} \right) - d \]

Flow Rule:
\[ M = \bar{\tau} + \beta \left( \frac{1}{3} \sigma_{kk} \right) \]

\[ s_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} \]
(Drained) elastic - plastic constitutive response
(pore pressure changes ignored)

**Elastic strain increment:**

\[
d^e \varepsilon_{ij} = \frac{ds_{ij}}{2G} + \frac{\delta_{ij}d\sigma_{kk}}{9K};
\]

here \( s_{ij} = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk} \).

**Plastic strain increment:**

\[
d^p \varepsilon_{ij} = \left( \frac{s_{ij}}{2\bar{\tau}} + \frac{\beta\delta_{ij}}{3} \right) \left( \frac{s_{kl}}{2\bar{\tau}} + \frac{\mu\delta_{kl}}{3} \right) \frac{d\sigma_{kl}}{h};
\]

here \( \bar{\tau} = \sqrt{\frac{1}{2}s_{ij}s_{ij}} \).

\[
d\varepsilon_{ij} = \frac{ds_{ij}}{2G} + \frac{\delta_{ij}d\sigma_{kk}}{9K} + \left( \frac{s_{ij}}{2\bar{\tau}} + \frac{\beta\delta_{ij}}{3} \right) \left( \frac{s_{kl}}{2\bar{\tau}} + \frac{\mu\delta_{kl}}{3} \right) \frac{d\sigma_{kl}}{h}.
\]
Localization and critical hardening required

The type of pressure dependent elastic-plastic material description used in these analyses has been shown by Rudnciki and Rice, [JMP, 1975] to permit localization of deformation for positive hardening, \( h \), particularly in deformation states near plane strain and predicted the critical hardening required to prevent localization of deformation for given stress state, \( \beta \), and \( \mu \). In plane strain, critical hardening is given by:

\[
\frac{h_{cr}}{G} = \frac{1 + \nu}{9(1-\nu)} (\beta - \mu)^2 - \frac{1 + \nu}{2} \left(\frac{\beta + \mu}{3}\right)^2
\]

[Templeton, Viesca & Rice, EOS, 2006, 2007]

Results show evidence of localization when \( h_{cr} \) is greater than zero, which cannot be resolved with increasing mesh refinement, but these features are suppressed with the addition of hardening above the critical level, \( h > h_{cr} \), to the elastic-plastic material description.
Poro-elastic-plastic constitutive response

Elastic strain increment:

\[ d^e \varepsilon_{ij} = \frac{d\sigma_{ij}}{2G} + \frac{\delta_{ij}}{3K} \left( \frac{d\sigma_{kk}}{3} + \alpha dp \right); \]

\[ \alpha = \text{Biot pore pressure factor } [ = 1 - \frac{K}{K_s} \text{ in simple cases}]. \]

Here \( s_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk}. \)

Plastic strain increment:

\[ d^p \varepsilon_{ij} = \left( \frac{s_{ij}}{2\bar{\tau}} + \frac{\beta \delta_{ij}}{3} \right) \left( \frac{s_{kl}}{2\bar{\tau}} + \frac{\mu \delta_{kl}}{3} \right) \frac{d\sigma_{kl} + \delta_{kl} dp}{h}; \]

Terzaghi effective stress often appropriate

[specific cases identified (Rice, ASCE - EMD, 1977)].

Here \( \bar{\tau} = \sqrt{\frac{1}{2} s_{ij} s_{ij}}. \)

Increment of fluid mass content:

\[ \frac{dm}{\rho_f} = \alpha \left( \frac{d\sigma_{kk}}{3} + \frac{dp}{B} \right) + d^p \varepsilon_{kk}; \]

\[ B = \text{Skempton factor } [ = \frac{1 - K / K_s}{1 - (1 + n)K / K_s + nK / K_f}, \text{ in smp. cs.}] \]

\[ d^p n = d^p \varepsilon_{kk} \text{ (specific cases: [Rice, ASCE - EMD, 1977])}. \]
Poro - elastic - plastic constitutive response

Elastic strain increment:

\[
d^e \varepsilon_{ij} = \frac{d s_{ij}}{2G} + \delta_{ij} \left( \frac{d \sigma_{kk}}{3} + \alpha d p \right); \]

\[\alpha = \text{Biot pore pressure factor} \left[ = 1 - \frac{K}{K_s} \right. \text{in simple cases}\right].\]

Here \( s_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk} \).

Plastic strain increment:

\[
d^p \varepsilon_{ij} = \left( \frac{s_{ij}}{2\tau} + \frac{\beta \delta_{ij}}{3} \right) \left( \frac{s_{kl}}{2\tau} + \frac{\mu \delta_{kl}}{3} \right) \frac{d \sigma_{kl} + \delta_{kl} d p}{h}; \]

Terzaghi effective stress often appropriate

[specific cases identified (Rice, ASCE - EMD, 1977)].

Here \( \tau = \sqrt{\frac{1}{2} s_{ij} s_{ij}} \).

Increment of fluid mass content:

\[
\frac{d m}{\rho_f} = \alpha \left( \frac{d \sigma_{kk}}{3} + \frac{d p}{B} \right) + d^p \varepsilon_{kk}; \]

\[B = \text{Skempton factor} \left[ = \frac{1 - K / K_s}{1 - (1 + n)K / K_s + nK / K_f} \right. \text{in smp. cs.}\right].\]

\[d^p n = d^p \varepsilon_{kk} \text{ (specific cases: [Rice, ASCE - EMD, 1977])}.\]

Form invariance, drained and undrained

Drained, \( dp = 0 \):

\[
d \varepsilon_{ij} = \frac{d s_{ij}}{2G} + \frac{\delta_{ij} d \sigma_{kk}}{9K} + \left( \frac{s_{ij}}{2\tau} + \frac{\beta \delta_{ij}}{3} \right) \left( \frac{s_{kl}}{2\tau} + \frac{\mu \delta_{kl}}{3} \right) \frac{d \sigma_{kl}}{h}. \]

Undrained, \( dm = 0 \):

\[
\frac{d m}{\rho_f} = \frac{d s_{ij}}{2G} + \frac{\delta_{ij} d \sigma_{kk}}{9K_u} + \left( \frac{s_{ij}}{2\tau} + \frac{\hat{\beta} \delta_{ij}}{3} \right) \left( \frac{s_{kl}}{2\tau} + \frac{\hat{\mu} \delta_{kl}}{3} \right) \frac{d \sigma_{kl}}{\hat{h}}, \]

where:

\[K_u = \frac{K}{1 - \alpha B} = \text{undrained bulk modulus},\]

\[\hat{h} = h + \mu \beta KB / \alpha,\]

\[\hat{\mu} = (1 - B) \mu, \quad \hat{\beta} = (1 - B) \beta.\]
Effects of $\Psi$, angle of most compressive stress, B, Skempton coefficient, and $\beta$, dilation

$\Psi = 14^\circ$: Pore Fluid Effects

$S = 1.0$  $\text{CF} = 0.88$  $\beta = 0.0$  $\mu = 0.51$  $f_s = 0.6$  $f_d = 0.1$

Initial Stress State

Proximity of Initial Stresses to Yield Surface

$R = \text{CF} \sigma_m \sin \phi$

Seismic S Ratio

$\tau_p = f_s \sigma_{yy}^0$

$\tau_y = f_d \sigma_{yy}^0$

$\varepsilon^p / (\tau_p/2\mu)$

$
\begin{array}{c}
\text{Equivalen Plastic Strain} \\
\text{Increases in pore pressure on the compressional side of the fault reduce the effective stress, increasing the likelihood of plastic deformation. For shallow angles of most compressive pre-stress, the presence of pore fluids increases the extent of inelastic deformation on the compressional side of the fault while decreasing its extent on the extensional side. The presence of dilatant plastic strains slightly increases the magnitude of equivalent plastic strain in the inelastically deforming region on the compressional side of the fault, but does not alter the rupture velocity or accumulated slip.}
\end{array}$

$\Psi = 56^\circ$: Pore Fluid Effects

$S = 1.0$  $\text{CF} = 0.49$  $\beta = 0.0$  $\mu = 0.60$  $f_s = 0.45$  $f_d = 0.045$

Initial Stress State

Proximity of Initial Stresses to Yield Surface

$R = \text{CF} \sigma_m \sin \phi$

Seismic S Ratio

$\tau_p = f_s \sigma_{yy}^0$

$\tau_y = f_d \sigma_{yy}^0$

$\varepsilon^p / (\tau_p/2\mu)$

$
\begin{array}{c}
\text{Equivalen Plastic Strain} \\
\text{Decreases in pore pressure on the extensional side of the fault increase the effective stress, decreasing the likelihood of plastic deformation. For steep angles of most compressive pre-stress, the presence of pore fluids decreases the extent of plastic straining on the extensional side. The presence of dilatant plastic strains slightly decreases the magnitude of equivalent plastic strain in the inelastically deforming region on the extensional side of the fault, but does not alter the rupture velocity or accumulated slip.}
\end{array}$
Dissimilarity of material properties across faults:
Poroelastic dissimilarity of saturated damage fringes at fault walls.

(Complementary to the effects of elastic dissimilarity of adjoining crust -- Weertman, Adams, Andrews & Ben-Zion, Harris & Day; the two effects may give the same or opposite "preferred" directions)

[Rudnicki & Rice, JGR, 2006; Dunham & Rice, EOS, 2006 & in progress]
For case shown:

\[ k^+ \beta^+ \gg k^- \beta^- \]

material 1

right-lateral slip \( \delta(t) \)

compression \( \varepsilon_{kk}^+(t) < 0 \)

extension \( \varepsilon_{kk}^-(t) > 0 \)

material 2

\( p_0^-(t) < 0 \)

\[ p_0^+(t) > 0 \]

\[ p(y, t) \sim \sqrt{\alpha_{\text{hy}} t} \]

\( k = \text{permeability} \)

\( \beta = \text{storage factor} \)
poroelastic favors; elastically identical (no elastic preferred direction)

Bottom side of fault is more damaged, much more permeable
Summary, some poromechanical processes in fault zones during earthquake rupture:

- Flash heating of microscopic asperity contacts:
  - Reduces frictional coefficient in rapid slip;
  - Promotes self-healing rupture mode.

- Thermal pressurization of pore fluid by shear heating:
  - Reduces effective normal stress $\sigma_n - p$.

- Fluid saturation within damaged fault border zones:
  - Affects Mohr-Coulomb plastic strain patterns;
  - Interacts back on rupture dynamics.

- Across-fault dissimilarity of poroelastic properties in saturated damage fringes:
  - Mode II slip can reduce effective normal stress $\sigma_n - p$;
  - Contributes to preferred directionality of rupture.