

Mathematical Analysis of Compaction Models in Fractured Sedimentary Basins

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ABSTRACT

In this communication we are concerned by the mathematical analysis of a compaction model in a sedimentary basin. In this 2-D modelling, the sedimentary rocks are considered as a porous medium, saturated with water. Thanks to the mass conservation law, the force balance, the Darcy's law, the Terzaghi's relation, the elastoplastic and the Kozeny-Carman laws, we obtain a coupled system of three non-linear partial differential equations, taking into account a moving boundary. The unknowns are the vertical effective stress, the sediment velocity and the water pressure.

INTRODUCTION

In this paper we consider that a porous medium can model a sedimentary basin. Furthermore, the sedimentation at the bottom of the ocean (i.e. at the top of the sedimentary basin) and the ocean's water mass induce a porosity deformation, therefore a fluid migration.

There are few mathematical publications on compaction models in sedimentary basins, and most of them concern only the numerical aspects [Fowler et al. 1999], [Ismail-zade et al. 1998], [Luo et al. 1998] or [Wangen et al. 1990].

In this work, we assume that the sediment deformation is due to the vertical compaction. Therefore, we only use the vertical effective stress in the modelling of the porosity deformation. This common hypothesis is not restrictive as observed in [Perez 1998].

In the sequel, we consider a water flow modelling in a 2-D fractured porous medium, elaborated by [Masson 199] at the Institut Français du Pétrole (I.F.P); and the mathematical analysis concerns a time-discretized version of this model.

MATHEMATICAL MODELLING

In this study, we consider a physical domain Ω , split into three open lipschitz connected sub-domains, denoted by Ω_1 , Ω_2 and Ω_3 .

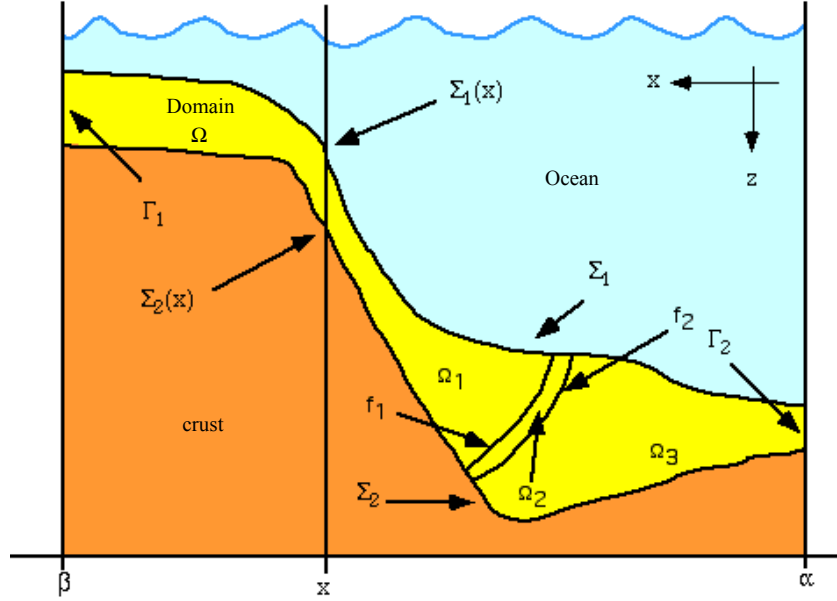


Figure 1: Domain

Ω_1 and Ω_3 are two sub-basins, separated by a fracture Ω_2 , with a small positive thickness (cf. fig.1).

Usually, for numerical simulations, one needs to assume that Ω_2 is an interface, that is, without any thickness. Therefore, one has to introduce some fracturation modelling on this interface, using empirical parameters (cf. [Masson 1999] or [Perez 1998]).

In our approach, we consider that Ω_2 is the same kind of domain as Ω_1 and Ω_3 . We only assume that the characteristic constants of the porous medium change with the sub-domain. Of course, an asymptotic study of the thickness of Ω_2 would give the same kind of results.

NOTATIONS

For any i in $\{s,w\}$ (w for water and s for sediments):

V_i is the velocity of phase i ,

ρ_i is the volumic mass of i ,

σ is the effective vertical stress,

σ_z is the total vertical stress,

ϕ is the porosity.

The porosity is assumed to be a function of the variables x , z and σ on Ω , and a decreasing function of the only variable σ on each Ω_i (different functions a priori),

K is the permeability tensor,

$F(\cdot)=[\rho_w\phi(\cdot)+\rho_s(1-\phi(\cdot))]g$,

p_w is the water pressure,

$${}^tB = (0, \rho_w g).$$

CONSERVATION LAWS

On each sub-domains, one has:

$$\begin{aligned} -\frac{\partial \phi(\sigma)}{\partial t} + \text{Div}[(1 - \phi(\sigma))V_s] &= 0 \\ \frac{\partial \phi(\sigma)}{\partial t} + \text{Div}[\phi(\sigma)V_w] &= 0 \\ \frac{\partial \sigma_z}{\partial z} &= [\rho_w \phi(\sigma) + \rho_s (1 - \phi(\sigma))]g \end{aligned} \quad (1)$$

BEHAVIOUR LAWS

Darcy's law:

$$\phi(\sigma)[V_w - V_s] = K(\nabla p_w - B) \quad (2).$$

Terzaghi's relation:

$$\sigma = \sigma_z - p_w \quad (3).$$

Vertical compaction hypothesis:

$${}^tV_s = (0, v_s)$$

Permeability law:

$$K = c \frac{\phi^3(\sigma)}{[1 - \phi(\sigma)]^2} \begin{pmatrix} \lambda_s & 0 \\ 0 & \lambda_a \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & k(\sigma, p_w) \end{pmatrix} \quad (4)$$

One may notice that K comes from the Kozeny - Carman tensor. k takes into account some subsiding of micro-fractures, once a certain threshold is reached in the physical system (cf. [Masson 1999] or [Gagneux Plouvier-Debaigt Vallet 2000]).

SYSTEM OF EQUATION

All the above relations lead us to consider, on each sub-domain, the following system of equations:

$$\begin{aligned} \frac{\partial \sigma}{\partial z} &= F(\cdot, \cdot, \sigma) - \frac{\partial p_w}{\partial z} \\ -\frac{\partial \phi(\cdot, \cdot, \sigma)}{\partial t} + \frac{\partial}{\partial z} [(1 - \phi(\cdot, \cdot, \sigma))v_s] &= 0 \\ -\text{Div}[K(\cdot, \cdot, \sigma, p_w)(\nabla p_w - B)] + \frac{\partial v_s}{\partial z} &= 0 \end{aligned} \quad (5)$$

DOMAIN AND BOUNDARY CONDITIONS

. The domain:

We assume that there exists some lipschitz functions γ_i and f_i , $i=1,2$, such that:

$$\begin{aligned}
 \Omega &= \{(x, z) \in R^2 / \alpha < x < \beta \text{ and } \gamma_1(x) < z < \gamma_2(x)\} \\
 \Omega_1 &= \{(x, z) \in \Omega / z < f_1(x)\}, \\
 \Omega_2 &= \{(x, z) \in \Omega / f_1(x) < z < f_2(x)\}, \\
 \Omega_3 &= \{(x, z) \in \Omega / f_2(x) < z\}, \\
 \Sigma_i &= \{(x, z) \in R^2 / \alpha < x < \beta \text{ and } y = \gamma_i(x)\} i=1,2, \\
 \Gamma_1 &= \{(\alpha, z) \in R^2 / \gamma_1(\alpha) < z = \gamma_2(\alpha)\}, \\
 \Gamma_2 &= \{(\beta, z) \in R^2 / \gamma_1(\beta) < z = \gamma_2(\beta)\}, \\
 f_i &= \{(x, z) \in \Omega / y = f_i(x)\} i=1,2,
 \end{aligned} \tag{6}$$

and we denote by n the unit outward normal on Γ , the boundary of Ω .

. The boundary conditions:

On Σ_1 , we assume that: (7)

$$\begin{aligned}
 p_w &= P_{\text{atm}} + \rho_s g H_w = P_{\Sigma_1} \\
 \text{where } H_w &\text{ is the ocean deepness,} \\
 \sigma &= 0 \quad \text{and} \quad v_s = g_s \\
 \text{where } g_s &\text{ is the sediment velocity at the bottom of the ocean.}
 \end{aligned}$$

On Σ_2 , we assume that: (8)

$$\begin{aligned}
 V_w \cdot n &= 0, \quad \text{thus} \quad -K(\nabla p_w - B) = -\phi(\sigma) v_s n_z \\
 \text{where } n_z &\text{ is the vertical component of } n.
 \end{aligned}$$

On Γ_i , we assume that: (9)

$$V_w \cdot n = 0, \quad \text{thus} \quad -K(\nabla p_w - B) = 0.$$

. The interfaces conditions:

On f_i , we assume that: (10)

$$\begin{aligned}
 \sigma_{\Omega_i} &= \sigma_{\Omega_{i+1}}, \quad v_{s, \Omega_i} = v_{s, \Omega_{i+1}}, \quad P_{w, \Omega_i} = P_{w, \Omega_{i+1}} \\
 \text{and} \\
 -K(\nabla p_{w, \Omega_i} - B) &= -K(\nabla p_{w, \Omega_{i+1}} - B)
 \end{aligned}$$

VARIATIONNAL CONTEXT AND HYPOTHESES

For a given g , we note:

$$\begin{aligned} V^g &= \left\{ u \in H^1(\Omega) / u_{\Sigma_1} = g \right\} \\ W &= \left\{ u \in L^2(\Omega) / \frac{\partial u}{\partial z} \in L^2(\Omega) \right\} \text{ and } W^g = \left\{ u \in W / u_{\Sigma_1} = g \right\} \quad (11) \\ Y_p &= \left\{ u \in L^2(\Omega) / \frac{\partial u}{\partial z} \in L^q(\Omega_i), q = \frac{2p}{p+2} \right\} \text{ and } X = Y_2. \end{aligned}$$

And we assume that:

- . ϕ is, on each sub-domain, a non-increasing lipschitzian function, with small variations. Moreover, there exist two constants, c and C , such that $0 < c \leq \phi \leq C < 1$,
- . h is a non-negative bounded function, uniformly lipschitzian with respect to (x, z) ,
- . $\exists p > 2, V^{P_{\Sigma_1}} \cap W^{1,q}(\Omega) \neq \emptyset$ and $g_s \in L^\infty(\Sigma_1)$ with $W^{g_s} \neq \emptyset$.

DISCRETIZATION OF THE SYSTEM

Let us note $h = \Delta t$ the discretization step and σ_0 , in W^0 , the value of σ at time $t = 0$. Then, we are looking for (σ, v_s, p_w) in $W^0 \times W^{g_{s,x}} V^{P_{\Sigma_1}}$, solution of the following implicit discretized problem:

$$\begin{aligned} \frac{\partial \sigma}{\partial z} &= F(\cdot, \cdot, \sigma) - \frac{\partial p_w}{\partial z} \\ \frac{\phi(\cdot, \cdot, \sigma_0) - \phi(\cdot, \cdot, \sigma)}{h} + \frac{\partial}{\partial z} [(1 - \phi(\cdot, \cdot, \sigma)) v_s] &= 0 \quad (12) \\ -\text{Div}[K(\cdot, \cdot, \sigma, p_w)(\nabla p_w - B)] + \frac{\partial v_s}{\partial z} &= 0 \end{aligned}$$

for the same boundary and interfaces conditions: (7) to (10).

More precisely, p_w is looked for as a weak solution, in the sense:

$$\begin{aligned} p_w &\in V^{P_{\Sigma_1}} \text{ and } \forall v \in V^0, \\ \int_{\Omega} \nabla v K(\cdot, \cdot, \sigma, p_w)(\nabla p_w - B) dx + \int_{\Omega} \frac{\partial v_s}{\partial z} v dx &= \int_{\Sigma_2} \phi(\sigma) v_s n_z v d\sigma \quad (13) \end{aligned}$$

EXISTENCE OF A SOLUTION

In order to prove the existence of a solution, we shall use the Schauder-Tikhonov fixed point theorem, in the separable Hilbert spaces context [Gagneux Madaune-Tort 1996].

Therefore, for a fixed p_w , we consider $\sigma(p_w)$ and $v_s(p_w)$ solutions of the first and the second equation of the system (12) respectively. Next, we shall note $S(p_w)$ the solution of the following problem:

$$\begin{aligned} S(p_w) \in V^{P_{\Sigma_1}} \text{ and } \forall v \in V^0, \\ \int_{\Omega} {}^t \nabla_v K(\cdot, \cdot, \sigma(p_w), p_w) (\nabla S(p_w) - B) dx + \int_{\Omega} \frac{\partial v_s(p_w)}{\partial z} v dx \quad (14) \\ = \int_{\Sigma_2} \phi(\sigma(p_w)) v_s(p_w) n_z v d\sigma \end{aligned}$$

which is a linear version of equation (13).

Since S is an application from $V^{P_{\Sigma_1}}$ onto $V^{P_{\Sigma_1}}$, one has just to prove that S admits a fixed point.

REGULARITY OF P_w

In order to prove the uniqueness of the solution of the system, we need to prove a better regularity of the solution. That is: the first derivatives of p_w are in L^p for a $p > 2$. The technique is well known, it consists in using a Poisson equation perturbation.

Firstly introduced by [Meyers 1963], used again by [bensoussan 1978] and [Necas 1989], this idea has been adapted by [Obelembia 1996] and [Gagneux 1998] for some diphasic flow modelling.

It is proved in [Gagneux – Plouvier-Debaigt – Vallet 2000] that:

$$\exists p > 2, p_w \in W^{1,p}(\Omega) \text{ and } \|p_w\|_{W^{1,p}(\Omega)} \leq C(p) \quad (15).$$

UNIQUENESS OF THE SOLUTION

We propose to prove the uniqueness of the solution by using a transposition method. This idea has been used by [Antontsev 1984] for the study of some diphasic filtration systems, and by [Oleinik 1959] for the Holmgren duality method. It consists in studying the existence of the solution of a dual problem, in order to prove the uniqueness of the primal problem.

Let us consider (σ_1, v_1, p_1) and (σ_2, v_2, p_2) two solutions, and consider the notations: $\bar{\sigma} = \sigma_1 - \sigma_2$, $\bar{v} = v_1 - v_2$, $\bar{p} = p_1 - p_2$ (16)

$$\begin{aligned}
\phi' &= \begin{cases} \frac{\phi(\dots, \sigma_1) - \phi(\dots, \sigma_2)}{\bar{\sigma}} & \text{if } \sigma_1 \neq \sigma_2 \\ \frac{\partial \phi}{\partial \sigma}(\dots, \sigma_1) & \text{if } \sigma_1 = \sigma_2 \end{cases} & F &= \begin{cases} \frac{F(\dots, \sigma_1) - F(\dots, \sigma_2)}{\bar{\sigma}} & \text{if } \sigma_1 \neq \sigma_2 \\ \frac{\partial F}{\partial \sigma}(\dots, \sigma_1) & \text{if } \sigma_1 = \sigma_2 \end{cases} \\
D_2 K &= \begin{cases} \frac{K(\dots, \sigma_1, p_1) - \phi(\dots, \sigma_2, p_1)}{\bar{\sigma}} & \text{if } \sigma_1 \neq \sigma_2 \\ \frac{\partial K}{\partial \sigma}(\dots, \sigma_1, p_1) & \text{if } \sigma_1 = \sigma_2 \end{cases} & & (17) \\
D_1 K &= \begin{cases} \frac{K(\dots, \sigma_2, p_1) - \phi(\dots, \sigma_2, p_2)}{\bar{p}} & \text{if } p_1 \neq p_2 \\ \frac{\partial K}{\partial p}(\dots, \sigma_2, p_2) & \text{if } p_1 = p_2 \end{cases}
\end{aligned}$$

Since these functions are lipschitzian functions, all these terms are bounded borelian functions.

Moreover, let us note $D_1(\sigma_2) = {}^t[\nabla p_1 - B]D_1 K$ and $D_2(p_1) = {}^t[\nabla p_1 - B]D_2 K$.

After subtracting the corresponding equations, we have to transpose the derivatives in order to obtain, for any (α, β, γ) in $X \times Y_p \times V^0$ (where p is given in the previous section) the following equation:

$$\begin{aligned}
& \int_{\Omega} \bar{\sigma} \left\{ \phi' \left(v_2 \frac{d\alpha}{dz} - \frac{\alpha}{h} \right) + \left(\frac{d\beta}{dz} + F \beta \right) + D_2(p_1) \nabla \gamma \right\} dx + \\
& \int_{\Omega} \bar{v} \left\{ (1 - \phi(\dots, \sigma_1)) \frac{d\alpha}{dz} + \frac{\partial \gamma}{\partial z} \right\} dx + \\
& \int_{\Omega} \bar{p} \left\{ \frac{d\beta}{dz} + D_1(\sigma_2) \nabla \gamma \right\} dx + \int_{\Omega} {}^t \nabla \gamma K(\dots, \sigma_2, p_2) \nabla \bar{p} dx + \\
& \sum_i \int_{f_i} \bar{\sigma} [v_2 (\phi'^+ \alpha^+ - \phi'^- \alpha^-) - (\beta^+ - \beta^-)] n_z d\sigma + \\
& \sum_i \int_{f_i} \bar{v} [(1 - \phi^+(\dots, \sigma_1)) \alpha^+ - (1 - \phi^-(\dots, \sigma_1)) \alpha^-] n_z d\sigma - \\
& \sum_i \int_{f_i} \bar{p} [\beta^+ - \beta^-] n_z d\sigma = \int_{\Sigma_2} \bar{\sigma} \{ \beta + \phi' v_2 (\alpha + \gamma) \} n_z d\sigma + \\
& \int_{\Sigma_2} \bar{v} \{ 1 - \phi(\dots, \sigma_1) \} (\alpha + \gamma) n_z d\sigma + \int_{\Sigma_2} \bar{p} \beta n_z d\sigma.
\end{aligned} \tag{18}$$

Where:

. ϕ^+, ϕ'^+ and ϕ^-, ϕ'^- denote the traces of ϕ, ϕ' on the right side and on the left side of f_i , by choosing the orientation of n , the normal vector, from Ω_i to Ω_{i+1} ,

. $\frac{d\alpha}{dz}$ (Resp. with β) denotes the absolutely continuous part with respect to the Lebesgue measure of the measure $\frac{\partial \alpha}{\partial z}$, since $\frac{\partial \alpha}{\partial z}$ is a bounded Radon measure for any α in X (Resp. β in Y_p).

That is to say, we have to find (α, β, γ) in $X \times Y_p \times V^0$, solution of the system:

$$\begin{aligned}
(1 - \phi(\cdot, \cdot, \sigma_1)) \frac{d\alpha}{dz} + \frac{\partial \gamma}{\partial z} &= 0 \\
\phi \left(v_2 \frac{d\alpha}{dz} - \frac{\alpha}{h} \right) + \left(\frac{d\beta}{dz} + F \beta \right) + D_2(p_1) \nabla \gamma &= 0 \\
\frac{d\beta}{dz} + D_1(\sigma_2) \nabla \gamma - \text{Div}[K(\cdot, \cdot, \sigma_2, p_2) \nabla \gamma] &= \bar{p}
\end{aligned} \tag{19}$$

with the interface conditions on f_i :

$$\begin{aligned}
(1 - \phi^+(\cdot, \cdot, \sigma_1)) \alpha^+ &= (1 - \phi^-(\cdot, \cdot, \sigma_1)) \alpha^- \\
v_2 (\phi^+ \alpha^+ - \phi^- \alpha^-) &= \beta^+ - \beta^-
\end{aligned} \tag{20}$$

and the boundary conditions:

$$\begin{aligned}
\gamma &= 0 \quad \text{on } \Sigma_1 \\
\beta &= 0 \text{ and } \alpha = -\gamma \quad \text{on } \Sigma_2 \\
-nK(\cdot, \cdot, \sigma_2, p_2) \nabla \gamma &= 0 \quad \text{on } \partial\Omega \setminus \Sigma_1
\end{aligned} \tag{21}$$

In order to prove the existence of such a solution, let us fix γ in V^0 .

Therefore, it is obvious to find $\alpha[\gamma]$ in X and $\beta[\gamma]$ in Y_p , respectively unique solutions of the first and of the second equation of (19) and plug them in the last one. Since, $\alpha[\gamma]$ and $\beta[\gamma]$ are linear continuous function of the variable γ , one has to solve a linear elliptic equation on V^0 .

. The LP property of the first derivatives of p_w and the sobolev embeddings allow the existence and the continuity of the bilinear form:

$$\begin{aligned}
\int_{\Omega} \frac{d\beta[\gamma]}{dz} v dx + \int_{\Omega} D_1(\sigma_2) \nabla \gamma v dx + \int_{\Omega} \nabla v K(\cdot, \cdot, \sigma_2, p_2) \nabla \gamma dx + \\
\sum_i \int_{f_i} (\beta^+[\gamma] - \beta^-[\gamma]) v n_z d\sigma = b(\gamma, v)
\end{aligned} \tag{22}$$

. The hypothesis of smallness of the variation of ϕ implies that this form is elliptic, from which the existence of a solution γ is obtained.

Using $(\alpha[\gamma], \beta[\gamma], \gamma)$ in (18), one has that $p_1 = p_2$ and then (σ, v_s, p_w) is the unique solution of (7) to (10) and (12) – (13).

A REMARK ABOUT THE CORRECTION OF THE UPPER FREE BOUNDARY

In this section, we propose one possibility to correct the free boundary.

Σ_1 is a free boundary as a consequence of the sedimentation, the erosion and the compaction effects. Using the hypothesis of vertical compaction and thanks to the volumic conservation law, one has, for any x of $] \alpha, \beta[$:

$$\int_{\Sigma_2(x)}^{z^*(x)} 1 - \tilde{\phi}(\cdot, \cdot, \sigma) dz = \int_{\Sigma_2(x)}^{\Sigma_1(x)} 1 - \phi(\cdot, \cdot, \sigma) dz + (Q_s n)_x h \tag{23}$$

where:

$(Q_s.n)_x$ represents a sedimentation or erosion modelling term at the bottom of the ocean,

$$\tilde{\phi}(\cdot, \cdot, \sigma) = \begin{cases} \phi(\cdot, \cdot, \sigma) & \text{if } z \leq \Sigma_1(x) \\ \phi(\cdot, \cdot, 0) & \text{if } z > \Sigma_1(x) \end{cases}$$

and

$z^*(x)$ is the new value of $\Sigma_1(x)$ since, for any x , $h_x : z^* \rightarrow \int_{\Sigma_2(x)}^{z^*} 1 - \tilde{\phi}(\cdot, \cdot, \sigma) dz$ is an increasing function.

A REMARK ABOUT THE LITHOLOGY

This academic study can be extended to a more complex geological situation (several fractures or different kind of sediments). The important thing is then to score the domain into enough sub-domains, with different geological or rheological characteristics.

REFERENCES

- Antontsev S.N. - Domansky A.V.: Uniqueness of generalized solutions of degenerate problem in two-phase filtration. *Numerical methods mechanics in continuum* Collection Sciences Research, Sbornik, **15**(6), pp. 15-28, 1984 (in Russian).
- Bensoussan A. - Lions J.L. - Papanicolaou G.: *Asymptotic analysis for periodic structures*, North-Holland, Amsterdam, 1978.
- Fowler, Andrew C. - Yang, Xin-she: Fast and slow compaction in sedimentary basins, *SIAM J. Appl. Math.* **59**(1), pp. 365-385, (1999).
- Gagneux G.: Sur l'analyse de modèles de la filtration diphasique en milieu poreux, in *Equations aux dérivées partielles et applications : Articles dédiés à J.L. Lions*, Gauthier-Villars, Elsevier, pp.527-540, 1998.
- Gagneux G. - Madaune-Tort M.: *Analyse mathématique de modèles non linéaires de l'ingénierie pétrolière*, Mathématiques & Applications n°22 , Springer-Verlag, 1996.
- Gagneux G.- Plouvier-Debaigt A. - Vallet G.: *Modélisation et analyse mathématique d'un écoulement 2D monophasique dans un bassin sédimentaire faillé sous l'effet de la compaction verticale*, publication interne du Laboratoire de Mathématiques Appliquées CNRS-ERS 2055, n°2000-31, 2000.
- Ismail-zade, A.T. - Korotkii, A.I. - Naimark, B.M. - Suetov, A.P. - Tsepelev, I.A.: Implementation of a three-dimensional hydrodynamic model for evolution of sedimentary basins, *Comput. Math. Math. Phys.* **38**(7), pp.1138-1151, 1998.

Luo X. - Vasseur G. - Pouya A. - Lamoureux-Var V. - Poliakov A.: Elastoplastic deformation of porous media applied to the modelling of compaction at basin scale, *Marine and Petroleum Geology*, **15**, pp.145-162, 1998.

Masson R.: personnel communication, IFP, 1999.

Meyers N.G.: An L_p -estimate for the gradient of solutions of second order elliptic divergence equations, *Ann. Sc. Norm. Sup. Pisa*, **vol.17**, pp.189-206, 1963.

Necas J.: *Ecoulements de fluide, Compacité par entropie*, Collection Recherche et Mathématiques Appliquées, n°10, Masson, 1989.

Obelembia Adande H.: *Contribution à l'étude de l'unicité pour des systèmes d'équations de conservation. Cas des écoulements diphasiques incompressibles en milieu poreux*, Thèse de l'Université de Pau, 1996.

Oleinik O.A.: Uniqueness and stability of the generalised solution of the Cauchy problem for a quasi-linear equation, *Uspehi Mat. Nauk*, **14**, p.165-170, 1959.

Perez L.: *Modélisation de la compaction dans les bassins sédimentaires: Influence d'un comportement mécanique tensoriel*, thèse de l'ENSAM, 1998.

Wangen, Magnus - Antonsen, Boerre - Fossum, Bjoern - Alm, Leif Kristian: A model for compaction of sedimentary basins, *Appl.. Math. Modelling* , **14**(10), pp. 506-517 (1990).

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