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HETEROGENEOUS DISPERSIVE HENRY PROBLEM

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EXTENDED ABSTRACT

The effect of heterogeneity on seawater intrusion is analyzed. Heterogeneity in hydraulic properties such as hydraulic conductivity is well known to significantly affect groundwater flow and solute transport. Yet, few studies [Mc Kribbin and O`Sullivan, 1980; Schincariol and Schwartz, 1990; Schincariol et al., 1997 and 1998; Simmons et al., 2001] have focused on evaluating the impact of heterogeneity on variable density flow. These works cannot be directly extended to saltwater intrusion problems, as they focus on instabilities caused by dense fluids overlying lighter ones and not on the opposite case. Our objective is to evaluate the effect of heterogeneity on the movement of the saltwater wedge and also on the thickness of the mixing zone.

We have adopted the well-known Henry Problem to assess the effect of heterogeneity on seawater intrusion because of its simplicity. However, the classical Henry Problem is purely diffusive. This would be unrealistic for simulating transport in heterogeneous media, as dispersion depends on the locally heterogeneous velocity field. Therefore, we have modified the traditional Henry problem by one where mixing is simulated by a dispersion term, instead of a diffusive one. We term it as the dispersive Henry problem. All parameters, but dispersivity, are the same as for the classical Henry problem. Dispersivity (α_L) is chosen so that the dispersion coefficient is comparable to molecular diffusion (D_m) in Henry Problem. That is, $\alpha_L = D_m \bullet \varepsilon / Q$, with Q total freshwater flow and ε porosity; and $\alpha_T = \alpha_L/10$. Even in the homogeneous case, the solution for the dispersive problem is markedly different from that of the traditional diffusive problem (Figure 1). Let us point out that by reducing dispersion in the locations where velocity is small (as is the case for the mixing zone), we obtain the relatively narrow mixing zones that are typically observed in real problems.

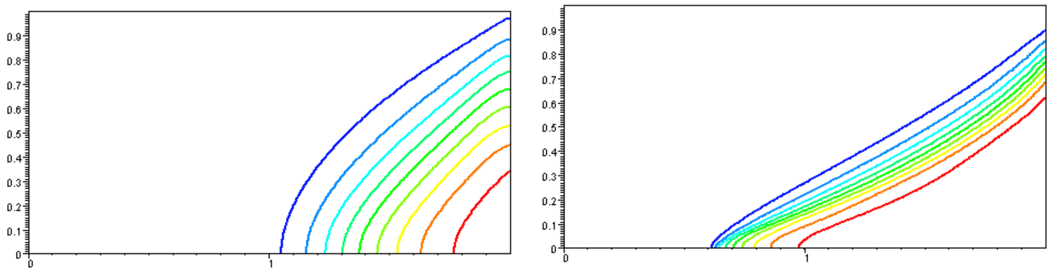

Figure 1: Comparison of traditional (left) and dispersive (right) Henry problem for the homogeneous case. Each curve corresponds to a percentage of salt content from 10% (top line; blue) to 90% (bottom line; red)

To assess the effect of heterogeneity, we simulated numerically saltwater intrusion in heterogeneous fields. The first step was to generate a suite of realizations of a multilognormal hydraulic conductivity (K) field using code GCOSIM3D [Gómez-Hernández, 1993]. Log-K fields are generated with a mean equal to the logarithm of the hydraulic conductivity used in Henry problem and unit variance. The input variogram is spherical with anisotropic correlation structure. The directional integral distances are 0.4 (λ_x) and 0.12 (λ_y).

SUTRA code [Voss, 1984] was employed in each of the simulations of density dependent flow and transport. The model domain was discretized in 200 x 100 quadrangular finite elements. Next we present the results for transport in ten such realizations. We compare them to those obtained in the simulations performed in effective homogeneous media. Effective homogeneous media are anisotropic, and the effective parameters (K_{effx}, K_{effy}) are calculated numerically. For the transport parameters we consider a suite of simulations with local dispersion, plus another one using macrodispersion values. The values of macrodispersion (A_L, A_T) were calculated using the stochastic analytical solutions by Gelhar and Axness (1983).

The positions of the 50% mixing line for all heterogeneous cases (blue lines) and the dispersive homogeneous Henry Problem solution (red line) are displayed in Figure 2. The variation is rather wide. Moreover, most of the 50% mixing lines penetrate further inland than that for a homogeneous medium with the same local dispersion. We see that the homogeneous case provides a poor approximation to describe saltwater intrusion in a heterogeneous case.

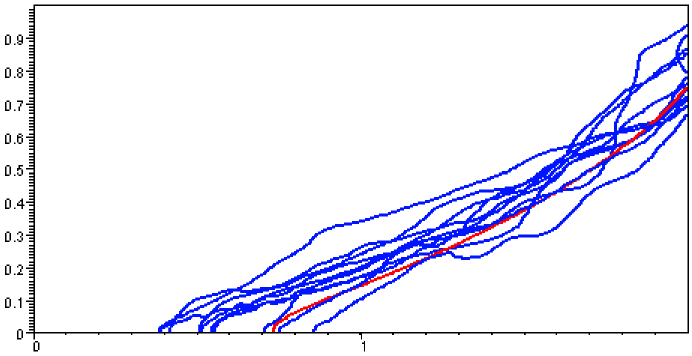

Figure 2: 50% mixing line for the heterogeneous (blue lines) and the homogeneous case with the same local dispersion (red line).

The large differences observed in Figure 2 can be partly attributed to the fact that the equivalent directional hydraulic conductivities are different in each simulation. In fact, when the solution for each realization is compared to its corresponding equivalent homogeneous case (Figure 3), the match is much better. However, they still differ significantly is some of the cases (Figure 4), especially in the upper and lower part of the interface. The solution is very sensitive to the local K distribution near the model boundaries. High permeability values at the upper part of the seaside boundary result in a thick saltwater wedge penetration in that area (figures 3b and 4b). On the other hand, low permeability values near the bottom boundary (Figure 4a) act as saline intrusion barriers and cause the toe of the saline wedge to penetrate less inland than in the corresponding homogeneous case. A general trend is observed in all simulations; the interface accommodates below the high hydraulic conductivity zones present in the vicinity of the homogeneous interface position.

In order to quantify the above observations, we define average mixing zone thickness as the area between the 25% and 75% mixing lines divided by the toe penetration (this one corresponding to the 50% mixing line). By comparing all the simulated cases, the heterogeneous ones plus the two homogeneous with local and macrodispersive values (Figure 5), we observe that the thickness of the mixing zone for the heterogeneous cases is almost always between the other two.

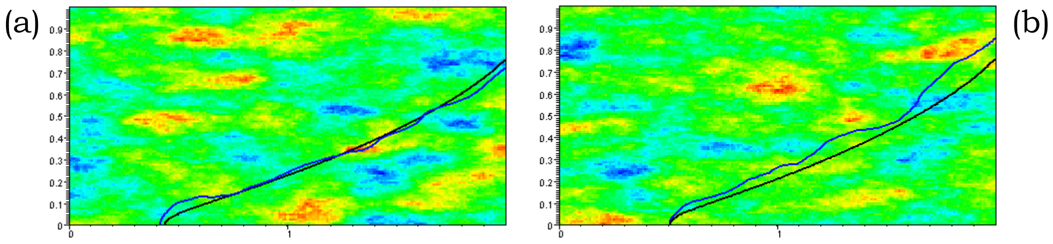

Figure 3: 50% mixing line for two heterogeneous (blue lines) and its corresponding effective homogeneous cases (black line).

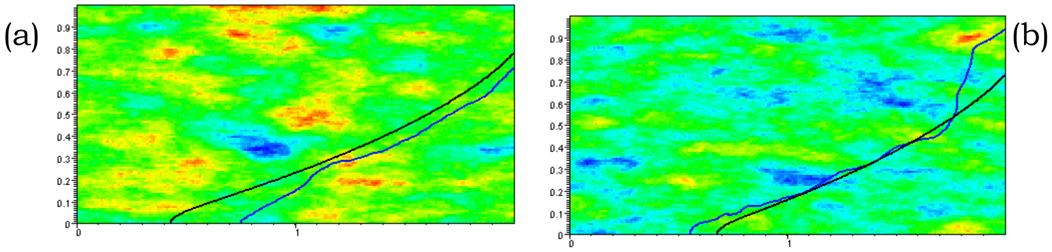

Figure 4: 50% mixing line for two heterogeneous (blue lines) and its corresponding effective homogeneous cases (black line). Observe how the toe separates from the effective homogeneous case position in *a*, and how the penetration is greater in the upper part of the seaside boundary in case *b*.

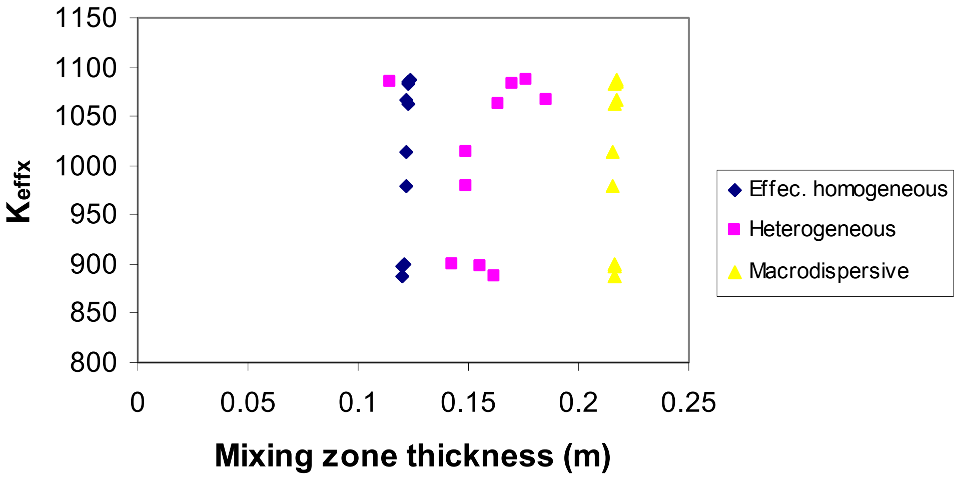

Figure 5: Mixing zone thickness vs. effective permeability in x direction. The thickness for the heterogeneous cases lies, except for one case, in between those obtained with local dispersion coefficients and the effective case with macrodispersion coefficients.

To sum up, saltwater movement in heterogeneous media is rather complex and is not clearly represented either by the homogeneous case or an effective homogeneous case. Only some approximations can be made about the interface position (50% mixing line), that is similar to the one of the effective homogeneous case except in the proximity of the model boundaries. There, the interface position is very affected by the local hydraulic conductivity distribution. The amplitude of the mixing zone is neither well represented by a homogeneous medium with either local or effective dispersivity, but it lies between these two extreme cases.

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