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Collocation method in solving saltwater intrusion problems

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EXTENDED ABSTRACT

In this work we present an improved numerical solution of the saltwater intrusion problem. Presented numerical model solves the nonlinear couple equations by the collocation method named **F**up **F**ragment **C**ollocation **M**ethod (FFCM) and uses R_{bf} basis functions of $Fup_2(x,y)$ type. Rvachev's basis functions - R_{bf} are infinitely derivable functions (B. Gotovac and Kozulic, 1999, 2000) classified between classical polynomials and spline functions, but in practice their application as basis functions is closer to splines. Thus, these functions can be treat as splines of infinite degree of the order that enables setting of the collocation points to the vertices of the basis fuctions. It enables also consistent, unique collocation method and application of the classical formulation and high approximation accuracy. The main property of this numerical method is universal vector space of R_{bf} basis functions that enables that solution with more collocation points (more basis functions) must be more accurately (it is not always the case, for instance, in the common finite element method). In that way we can improve our solution for any prescribed tolerance or degree of the accuracy. It can be using for creating "accurate" solution of the known saltwater intrusion problem as the Henry problem (H. Gotovac et al., 2003) and can be use as benchmark for other numerical codes.

In this extended abstract we would like to present main characteristics of the presented method and its application to the saltwater intrusion problem. The method solves two nonlinear coupled equations with two unknowns; pressure, p, and concentration, c. Velocity field is evaluated at the collocation nodes using Darcy law. Thus, we avoided classical numerical problems present with the finite elements method. The solution is found in the form of functions p and c and all their derivatives like specific flux q. There is no classical numerical integration based on spatial discretization but FFCM solves governing equations directly and only number and location of collocation nodes is changed.

All boundary conditions are provided similar to the classical problems (Bear, 1999; Voss and Souza, 1987), which are frequently used in literature. Particular attention is given to specifying sea boundary condition which describes the interaction on the border between the sea and the coastal aquifer. Generally, the choice of specifying sea boundary condition as a mixed type is commonly used but has a disadvantage of *a priori* fixing outflow and inflow portions at the sea bottom. This outflow portion represents the submarine groundwater discharge (SGD) zone that has a strong environmental impact on the coastal waters. The use of Cauchy type of boundary condition with a buffer zone is appealing from the point of calculating exactly the outflow portion at the sea bottom but has disadvantage of introducing the parameter in the buffer zone that has to be calibrated either with available data or with some previous solutions.

The FFCM is applied using the hierarchical increase of the collocation nodes only in the sensitive zones within the domain. The solution is sought by first solving the entire domain using a coarse grid of collocation nodes to assure a correct mass balance and stable solution, then additional collocation nodes are added in the sensitive zones (sources/sinks, transition zones, outflow region) to improve the solution accuracy until predefined numerical criteria are satisfied. The final solution is verified using classical benchmark problems (Henry, Elder, Hydrocoin …) for variable density flow in homogeneous domain. However, the challenge is the introduction of FFCM to the heterogeneous aquifer and solving the submarine groundwater discharge (SGD) problem which is becoming very important from the point of land-based sources of pollution.

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