

Three Dimensional Mixed Finite Elements Methods and High Resolution Finite Volumes for the Solution of Density Dependent Flow and Transport in Groundwater

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The density dependent flow and transport problem in groundwater on two dimensional triangular grids has been solved numerically by means of a mixed finite element scheme for the flow equation and a mixed finite element-finite volume time-splitting based technique for the transport equation. This procedure has been shown to be an effective tool even when the process is dominated by the effects of density flow if solute concentration gradients are high. In this work we develop and study the extension of this approach to three dimensions employing tetrahedral elements. The Mixed Hybrid Finite Element (MHFE) method is used for the flow equation and a time splitting technique for the transport, combining the MHFE method with a High Resolution Finite Volume (HRFV) stencil. Implementation in time is allowed using an explicit time-stepping for HRFV and an implicit time-stepping for MHFE. The extension of this technique to three dimensions requires careful consideration on the adoption of an appropriate HRFV scheme combined with a truly three-dimensional limiter that preserves second order accuracy in space. An efficient strategy is developed by applying a second order linear reconstruction using the least squares method with a limiting procedure based on the Barth-Jespersen slope limiter. Three dimensional benchmark tests, such as the Elder problem extended to three dimensions by Diersch and Kolditz and the more recent saltpool problem introduced by Oswald, are performed in order to verify the accuracy and reliability of this numerical technique and discuss some critical aspects and challenges related to density-dependent problems.

Keywords

Porous media, mixed hybrid finite elements, high resolution finite volumes, time splitting, tetrahedral mesh

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