Voronoi percolation in $\mathbb{R}^d$

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Abstract

Place points $P_i$ randomly in $\mathbb{R}^d$ according to a uniform Poisson process and divide $\mathbb{R}^d$ into Voronoi cells $R_i = \{ P \in \mathbb{R}^d : d(P, P_i) \leq d(P, P_j) \text{ for all } j \}$ according to which point $P_i$ of the Poisson process a point $P$ is closest to. We say two Voronoi cells are adjacent if they share a common $(d-1)$-dimensional face. We consider two related questions.

1. Declare each Voronoi cell to be open independently with probability $p$. For what values of $p$ do there exist infinite connected regions of open cells?

2. On average, how many neighbors does a Voronoi cell have?

For the first question we show that there is a critical probability $p_c$ above which there will exist infinite clusters and below which all clusters are almost surely finite. Moreover, there exist absolute constants $C_1, C_2 > 0$ such that

$$C_1 (d \log d)^{-1} 2^{-d} < p_c < C_2 \sqrt{d} \log d 2^{-d}.$$ 

For the second question we show that the average number of neighbors is bounded above and below by expressions of the form $C d 2^d$. 

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