Abstract

It has been known for some time that there is a connection between linear codes over fields, and matroids. The connection was explored by Curtis Greene in a 1976 paper. In fact a generator matrix for a linear code over a field is also a representation of a matroid over that field. There are intimately related operations of deletion, contraction and duality on both the code and the matroid; the weight enumerator of the code is an evaluation of the Tutte polynomial of the matroid; and a standard identity relating the Tutte polynomials of dual matroids gives rise to a ‘MacWilliams identity’ relating the weight enumerators of dual codes.

We generalize all those results and operations to linear codes over rings. We define a weight enumerator for a code over a ring. We associate a type of polymatroid with a linear code over a ring, and define a type of Tutte polynomial for it. The standard relation between the weight enumerator and the Tutte polynomial is generalized. We define polymatroid duality, and in the case that there is a Morita duality, we define code duality, relate it to polymatroid duality, and find a ‘MacWilliams identity’.