On non-$z$($\text{mod } k$) dominating sets

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Abstract

For a graph $G$, a positive integer $k, k \geq 2$, and a non-negative integer with $z < k$ and $z \neq 1$, a subset $D$ of the vertex set $V(G)$ is said to be a non-$z$($\text{mod } k$) dominating set if $D$ is a dominating set and for all $x \in V(G), |N[x] \cap D| \neq z(\text{mod } k)$.

For the case $k = 2$ and $z = 0$, it has been shown that these sets exist for all graphs. The problem for $k \geq 3$ is unknown (the existence for even values of $k$ and $z = 0$ follows from the $k = 2$ case.) It is the purpose of this paper to show that for $k \geq 3$ and with $z < k$ and $z \neq 1$, that a non-$z$($\text{mod } k$) dominating set exists for all trees. Also, it will be shown that for $k \geq 4, z \neq 1, 2$ or 3 that any unicyclic graph contains a non-$z$($\text{mod } k$) dominating set. We also give a few special cases of other families of graphs for which these dominating sets must exist.