Mixed Hybrid Finite Element and Finite Volume for Flow in Porous Media

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ABSTRACT

Multiphase flow of fluids in porous media is physically and chemically complex. It involves heterogeneities in the porous media at many different length scales and complicated processes such as diffusion and dispersion. It is well known that the transport term in the flow equations are governed by fluid velocities. Thus accurate numerical simulation requires accurate approximations for these velocities. Often the flow properties of the porous media change abruptly with sharp changes in lithology. Consequently, the coefficient in the flow equation is quite rough.

The purpose of this paper is to discuss the applicability of the mixed finite element methods and finite volume methods to various types of flow in porous media. These problems occur in saltwater intrusion, in groundwater contaminant transport, and petroleum reservoir simulation. We focus on two-phase immiscible flow in heterogeneous porous media. The equations governing these types of flow can be effectively rewritten in a fractional flow formulation; i.e., in terms of a global pressure and saturation as the primary variables. This formulation leads to a coupled system of partial differential equations which includes a nonlinear degenerate parabolic saturation equation and an elliptic pressure-velocity equation. The saturation equation is convection dominated and thus special care should be taken in discretization. The diffusion term there is small but important and can not be neglected.

A numerical scheme based on the mixed hybrid finite element method for the pressure-velocity equation and a finite volume method for the saturation equation is developed. It is shown that this approach leads to robust schemes applicable for unstructured grids and the approximate solution has various interesting properties which correspond to the properties of the physical solution. Numerical simulations from two space dimensions tests on unstructured grid will be presented.

Keywords: Mixed hybrid method, finite volume method, heterogeneous porous media, unstructured grids.

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