Chance Constrained Optimal Management in Saltwater-Intruded Coastal Aquifers Using Genetic Algorithms

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Abstract

A chance-constrained management model with an economic objective of coastal aquifers threatened by saltwater intrusion is developed. The model explicitly considers that the toe location of saltwater intrusion is a function of random variables such as physical parameters and boundary conditions. This model incorporates the quantity, quality and economic aspects of groundwater for sustainable use of the coastal aquifers and extends the existing deterministic model to stochastic one. The management objective allows for a plausible scenario for economic planned withdrawal and salinity control in coastal aquifers. The steady-state model simulates groundwater flow on a horizontal plane using sharp-interface analytical approach. This model is incorporated into a simple genetic algorithm (SGA) and chance constrained optimization. The chance constraint is divided into an expected value component and a stochastic one. Both the expected value and stochastic component were evaluated using the perturbation technique based on the second order Taylor series approximation. When the input physical parameters and boundary conditions are given as mean and standard deviation, the solutions, either the toe location of the saltwater wedge or the entire interface location, are predicted as a mean and standard deviation. The chance constrained programming method transforms the probabilistic model to a deterministic one and therefore facilitate the solution and reduces memory requirements and computational time. The management model is illustrated through a hypothetical unconfined coastal aquifer system. The results show that incorporating uncertainty into coastal aquifer optimization model using chance constraint programming coupled to SGA could be a practical method for making decisions on optimal pumping rates and scenario exploitation schemes.

1 INTRODUCTION:

In coastal areas, several wells have been drilled to satisfy the increasing by water demand, which has caused unacceptable drawdowns and deterioration of water pumped quality. A set of well-established withdrawal and management policies is necessary to achieve more efficient management and operation of these aquifers.

Initial efforts to support and improve the development and operation of groundwater systems by simulation and optimization techniques were made in the early 1970s. Since then, various types of groundwater management models have been proposed and successfully applied to real-world aquifer systems. Many reviews on the types of groundwater management models and their applications are made by [Gorelick, 1983], and [Yeh, 1986]. The management models applications in saltwater intrusion, is relatively recent, [Cummings and McFarland, 1974], [Shamir, et al., 1984], [Willis and Finney, 1988], [Barlow, Wagner and Belitz, 1996], [Emch and Yeh, 1998], [El Harrouni et al., 1998],[Das and Datta, 1999], [Cheng et al, 1999], [Loaiciga and Leipnik, 2000]. Most of these problems have been investigated in a more complex setting involving various management objectives. Concerning saltwater intrusion into wells, it is often addressed in an indirect manner such as constraining drawdown at control points, or minimizing the intruded saltwater volume. These studies were
conducted to maintain aquifer levels and prevent the saltwater intrusion so that undesirable economic consequences and legal violations are prevented.

In this paper we deal with the development and application of an operational groundwater management model under conditions of uncertainty for a coastal aquifer system. Optimal management decisions aim to maximize the relative net benefit from allocation of the groundwater supplies, while minimizing well interference effects and the invasion of saltwater front to the wells. The optimal decisions of the model define the expected water targets for the principal agricultural, municipal and industrial demand centers and their greatest net economic benefit.

For simplicity and feasibility demonstration purposes only the sharp-interface saltwater intrusion model is used, particularly; the single-potential formulation of [Strack, 1976] is adopted for solving boundary value problems. A Simple Genetic Algorithm (SGA) [Goldberg, 1989] is used for optimization purposes. The SGA is based on three specialized genetic operators: selection, crossover, and mutation and uses a conventional binary string to represent the design variables, i.e. the pumping rates. However, regardless of the sophistication of the deterministic coastal aquifer management model, it is more realistic to consider the stochastic nature of coastal aquifer parameters including boundary conditions. In reality, some parameters such as hydrogeological (hydraulic conductivity, aquifer geometry) and hydrological (precipitation recharge, groundwater outflow, sea level) conditions and unknown anthropogenic operation conditions (pumping extraction, artificial recharge) are usually known with uncertainty and then considered as random parameters. They can only be estimated as a mean value and standard deviation.

The stochastic formulation of coastal aquifer management optimization model can be adopted in order to investigate the effect of random input data on the location of seawater/freshwater interface which can give information about the interval of confidence of the output concerning the toe or interface. Almost all parameters such as freshwater discharge, the recharge and well pumping rate are not known with certainty. The measurements include many types of errors, and hence, the stochastic approach required for realistic predictions. The Chance Constrained Programming (CCP) method [Charnes and Cooper, 1959] was applied to account for uncertainties arising in the coastal aquifer management model.

The SGA-chance-constrained optimization model identifies the greatest net economic benefit under the constraint of no intrusion of saltwater front to the wells is met at a prescribed reliability level. This model extends the existing deterministic model and incorporates measures of reliability in the prediction of the toe location. The model is subject also to constraints of discharge bounds. The uncertainty is represented in terms of statistical measures of mean, standard deviation and covariance and its analysis is based on the perturbation technique using Taylor series expansion [Cheng and Ouazar, 1995], [Naji et al, 1998].

Through only a hypothetical example is used , the SGA-CCP approach is quite efficient. The results show that the optimal pumping pattern and their relative net benefit decrease with increasing values of reliability levels.

2 STRACK BASED PUMPING SOLUTION.

In saltwater intrusion, [Strack, 1976] has given a single potential formulation that leads to 2-dimensional horizontal Laplace equation valid for both freshwater and saltwater zones.

\[ \nabla^2 \Phi = 0 \] (1)
The potential $\Phi$ takes the following definitions:

For confined aquifer:

$$\Phi = Bh_f + \frac{(s-1)B^2}{2} - sBd \quad \text{freshwater}$$

$$= \frac{1}{2(s-1)}[h_f + (s-1)B - sd]^2 \quad \text{saltwater}$$

(2)

for unconfined aquifer:

$$\Phi = \frac{1}{2}[h_f^2 - sd^2] \quad \text{freshwater}$$

$$= \frac{s}{2(s-1)}(h_f - d)^2 \quad \text{saltwater}$$

(3)

In the above $h_f$ is the freshwater piezometric head, $d$ is the elevation of mean sea level above the datum, $B$ is the confined aquifer thickness, $s$ is the saltwater and freshwater density ratio.

The problem is solved as a one-zone problem with appropriate boundary conditions. Once the problem is solved by analytical or numerical means, the interface location $x_i$ is evaluated as:

$$\xi = \sqrt{\frac{2\phi}{s-1}} + d - B \quad \text{For confined aquifer}$$

$$\xi = \sqrt{\frac{2\phi}{s(s-1)}} \quad \text{For unconfined aquifer}$$

(4)

The toe of saltwater wedge is located at $\xi = d$. From (4), this means that the toe is located at where $\Phi$ takes these values:

$$\Phi_{\text{toe}}^c = \frac{s-1}{2}B^2 \quad \text{For confined aquifer}$$

$$\Phi_{\text{toe}}^u = \frac{s(s-1)}{2}d^2 \quad \text{For unconfined aquifer}$$

(5)

For both the confined and unconfined aquifers, once the solution is found, the location of the toe can be tracked using the above equations.

The toe location $x_{\text{toe}}$ can be solved from:

$$\Phi_{\text{toe}} = \frac{q}{K}x_{\text{toe}} + \sum_{w=1}^{n} \frac{Q_w}{4\pi K} \ln \left( \frac{(x_{\text{toe}} - x_w)^2 + (y_{\text{toe}} - y_w)^2}{(x_{\text{toe}} + x_w)^2 + (y_{\text{toe}} - y_w)^2} \right)$$

(6)

Where $(x_w, y_w)$ are well coordinates, $Q_w$ is the pumping rate of well $w$ located at distance $x_w$ from the coast, $K$ is the hydraulic conductivity and $q$ is the freshwater outflow of rate.

3 GENETIC ALGORITHMS.

Nowadays, GAs are recognized as powerful search algorithms and offer nice alternative to conventional optimization technique. They are based on the mechanics of natural selection and natural genetics [Goldberg, 1989]. Its implementation starts with randomly generated set of coded strings representing potential solutions of the problem. Within an evolution iteration three basic genetic operators are applied to produce stronger
individuals for simple genetic process, which are: selection, crossover and mutation. The length of the string depends on the number of parameters to be optimized, the number and types of constraints, and the precision requirement of the final solution.

From the initial population, the fittest strings, as measured by their objective function values, are selected to pass their ‘genetic information’ to the next generation. This operation, called selection, mimics the survival of the fittest inherent in natural systems. There are many different schemes for selecting survivors. All share the common goal that fit members replace less fit members in the population to advance the search.

After selection the population is, on average, more fit than it was before selection. Then crossover creates from the survivors entirely new strings that contain various combinations of the traits that gave the survivors their fitness. The two parents are crossed if the generated number is less than the specified crossover probability fixed at the beginning of the process. The crossover point is also chosen randomly in the parents. The genes on one side of it are assigned to one child, while genes on the other side go to the other child.

The mutation is the last operator to be applied. It is typically performed by randomly altering the value of the selected gene. In the binary coded genes, mutation is typically performed by flipping the value of the 0/1 bit of each gene that is selected. The probability of selecting a gene for mutation is controlled by the mutation probability, which is usually set equal to a small value.

The purpose of mutation is to keep the population diverse, and to prevent the SGA from prematurely converging to a local optimum. The cycle continues until a new population is formed. Iteration then continues for the next generation of solutions. The process is considered as terminated when convergence is detected or when the specified maximum number of generations is reached. As the maximum value of the objective function value is not known only a maximum number of iterations are specified to stop the process and the best chromosome within these iterations can be selected as the optimum solution of the problem.

For most real-world problems, these pseudo-optimal solutions are still much better than what could be obtained using less robust methods. The SGA must have some control parameters such as population size, \(n\)-usually 20 to 100, and probabilities for applying genetic operators, e.g. crossover probability \(p_c\) - usually 0.7 to 1, and mutation probability \(p_m\) – usually 0.01 to 0.05.

In the pumping management problem, the design variables are the pumping rates and are encoded as binary string within the well capacity constraints. Each population will contain strings to represent all these design variables. SGA are ideally suited for unconstrained optimization problems. As the present problem is a constrained one, it is necessary to transform it into an unconstrained problem. Transformations are achieved by either using exterior or interior penalty functions. A formulation using exterior penalty functions is proposed in this paper.

4 REVIEW OF CHANCE CONSTRAINED GROUND-WATER MODELS.

[Charnes and Cooper, 1962] studied chance constrained programming by solving the deterministic equivalent of a stochastic optimization problem. The chance constraints can easily incorporate reliability measures imposed on the decision variables. A number of chance constrained models have been proposed for solving different problems of groundwater management. [Tung, 1986] developed a chance-constrained model for groundwater-quantity
management. This model takes into account the random nature of transmissivity and storage coefficient. It uses the first-order uncertainty analysis for the hydraulic conductivity and the storage coefficient. However, this model is not applicable to groundwater quality management. Specific applications of chance–constrained programming in aquifer management are proposed by [Wagner and Gorelick, 1987], [Hantush and Marino, 1989], [Morgan et al., 1993], [Ritzel et al. 1994], [Chan, 1994 ], [Datta and Dhiman, 1996], [Sawyer and Lin ,1998],[Wagner, 1999].

[Wagner and Gorelick, 1987] presented a modified form of the chance constrained programming to determine a pumping strategy for controlling groundwater quality. [Hantush and Marino, 1989] developed a chance-constrained model for stream-aquifer interaction. [Morgan et al., 1993] developed a mixed-integer chance-constrained programming. They demonstrate the applicability of the technique to groundwater remediation problems. Chance-constrained ground-water management models have been applied to design ground water hydraulic [Tiedman and Gorelick, 1993] and ground-water quality [Gailey and Gorelick, 1993] management strategies. The chance-constrained method for remediation design has been demonstrated in “ real world” systems. [Chan, 1994] developed a partial infeasibility method for chance-constrained aquifer management that uses heuristic methods requiring the design reliability level to be satisfied for a training set consisting of multiple realizations of uncertain transmissivity. [Datta and Dhiman, 1996] developed a chance –constrained model for designing a groundwater quality monitoring network.[Wagner, 1999] developed a chance-constrained model for identifying the least cost pumping strategy for remediating the groundwater contamination in the presence of simulation model uncertainty. This model is combined with the network design model for identifying the groundwater sampling strategy with maximum information contents.

Previous research on chance-constrained groundwater management models dealt with uncertainty in the constraint coefficients. The work of [Sawyer and Lin, 1998] extends previous studies by considering the combination of uncertainty in the cost coefficients and constraints of the groundwater management model. This paper provides a development of the chance constrained coastal aquifer management problem to account for uncertainty that could arise in coastal aquifer management model. A chance constrained-SGA optimization model is proposed for identifying the greatest net economic benefit for coastal aquifer management under uncertainty conditions.

5 FORMULATION OF OPTIMIZATION PROBLEM.

5.1 DETERMINISTIC MANAGEMENT MODEL.

The coastal aquifer management model is developed for sustainable water withdrawal from the aquifer for beneficial uses. Due to application of spatially distributed pumping strategy, the seawater intrusion takes place and the salinity of the pumped water varies with the magnitude and location of pumping in the two-dimensional space domain of the aquifer. The management model is developed considering a plausible objective function taking into account economic values. Explicit consideration of economic values requires the definition of cost and benefit functions. This aspect is included in the model. The pumping costs are assumed to be directly proportional to the product of the pumping rate and the total lift at each well. The mathematical formulation of the optimization problem can be written as:

\[
\text{Maximize } Z = \sum_{i=1}^{n} B_i Q_i - C_i Q_i (L_i - h_i)
\]
with respect to \( Q_i \) and other design variables. \( Z \) is the objective function, subject to the following constraints:

The management aspects are satisfied:

1. Toe location constraints:
   \[ X_{\text{toe},i} < X_i \quad i=1,n \]  
   (8)

2. The practical pumping capacity limits should not be exceeded:
   \[ Q_i^{\text{min}} \leq Q_i \leq Q_i^{\text{max}} \quad i=1,n \]  
   (9)

In the above, \( Q_i \) is the discharge rate of well \( i \), \( X'_w \) is the distance of well \( i \) from the coast, \( X_{\text{toe},i} \) is the toe location from the coast in front of well \( i \), \( Q_i^{\text{min}} \) and \( Q_i^{\text{max}} \) are respectively the minimum and maximum discharge of a well constrained by equipment capacity, \( h_i \) is the hydraulic head at well point \( i \), \( L_i \) initial lift at well \( i \), \( B_p \) is the benefit per unit supply of water at well point \( i \), \( C_p \) is the cost of pumping a unit volume per unit head at well point \( i \), and \( n \) is the number of wells. For existing wells at fixed locations, the design variables are the pumping rates \( Q_i \). For new wells to be installed, the design variables can include the number of wells, their discharges, and well locations in terms of \( x \)- and \( y \)-coordinates. The main constraint of the problem is no invasion of the wells criteria. Other constraints may include the maximum and minimum pumping rate of each well has to be satisfied.

To allow a genetic algorithm to be used, the constrained problem in (7) is first transformed into an unconstrained one. This is accomplished by imposing penalty for the constraints in the objective function. For example, to ensure no saltwater intrusion, a benefit function can be defined as:

\[
F = \sum_{i=1}^{n} B_p Q_i - C_p Q_i (L_i - h_i) - r \sum_{j=1}^{m} \left( \frac{1 - X_{\text{toe},j}}{X_{w,j}} \right)^2
\]  
(10)

Where \( r \) is the penalty factor and \( m \) is the number of violated constraints. If the constraints are violated, we assume that the benefit function is defined as:

\[
F = \sum_{i=1}^{n} B_p Q_i - C_p Q_i (L_i - h_i) - r \sum_{i=1}^{n} B_p Q_i^{\text{max}} - C_p Q_i^{\text{max}} (L_i - h_i)
\]  
(11)

In this case, the unconstrained maximization problem is solved.

**5.2 CHANCE–CONSTRAINED MANAGEMENT MODEL.**

A chance constrained optimization model of coastal aquifer management is formulated to account for uncertainties in the model coefficients. The chance constrained programming method is used to transform the probabilistic constrained model to the deterministic one. Solving the deterministic equivalent problem is much easier than the probabilistic problem. The formulation of the stochastic simulation-optimization problem begins with the deterministic formulation, but considers the dependent variables \( X_{\text{toe},i} \) which is described by a probability distribution at any location \( i \). The probability must be greater than some prescribed reliability, \( r_{\text{toe},i} \).

\[
\Pr\{X_{\text{toe},i}|X_i\} \geq r_{\text{toe},i}
\]  
(12)
Consequently, the left-hand side of the toe location constraints is random in nature because it contains random variables of freshwater outflow rate and pumping well discharge. This implies that the compliance of constraints at each pumping well can not be ensured with certainty. Thus, it is more appropriate and realistic to examine the constraint performance probabilistically. In the stochastic case, it is operationally feasible to specify limitations on allowable risk or required reliability of constraint performance. A probabilistic statement of toe location constraint in (12) is not mathematically operational and further modification or transformation is required. To make (12) mathematically operational, it is necessary to assess statistical properties of random terms in chance-constrained equations.

The chance constraint for a toe location is:

\[
E[X_{toe}^i] + N^{-1}(r_{toe}) \sigma[X_{toe}^i] \leq X_i
\]  

(13)

The first term is the expected value of the toe location and is the mean of the toe location. The second term is the stochastic component, where \( N^{-1} \) is the value of the standard normal cumulative distribution corresponding to a specified level \( r_{toe} \), and \( \sigma[X_{toe}^i] \) is the standard deviation of the toe location. This term reflects the uncertainty in the toe location due to the fact that the random variables freshwater outflow rate and pumping well discharge are unknown. If the toe location is known and equal to the estimated value, then the stochastic component is zero.

In order to obtain the statistical information required in chance constraints, a relationship must be obtained between the uncertain model parameter values and dependent variables. In present work, we assume that the perturbation of a random variable from the mean is a small quantity. Taylor’s series can be employed to find approximate expressions of statistical moments [Cheng and Ouazar, 1995], [Naji et al, 1998].

The mean and variance of the toe location are given by:

\[
X_{toe} = X_{toe} + \frac{1}{2} \left[ \frac{\partial^2 X_{toe}}{\partial q^2} \sigma_q^2 + \frac{\partial^2 X_{toe}}{\partial Q^2} \sigma_Q^2 \right] + 2 \frac{\partial^2 X_{toe}}{\partial q \partial Q} \sigma_{qQ} \]  

(14)

\[
\sigma^2_{X_{toe}} = \left[ \frac{\partial X_{toe}}{\partial q} \right]^2 \sigma_q^2 + \left[ \frac{\partial X_{toe}}{\partial Q} \right]^2 \sigma_Q^2 + 2 \frac{\partial X_{toe}}{\partial q} \frac{\partial X_{toe}}{\partial Q} \sigma_{qQ} \]  

(15)

The covariance \( \sigma_{qQ} \) can be set to zero based on physical considerations, and the result can be somewhat simplified.

In the above, \( X_{toe} \) is solved by strack pumping well solution:

\[
\Phi_{toe} = \frac{q}{K} x_{toe} + \sum_{j=1}^{n} \frac{Q_w}{4\pi K} \ln \left[ \frac{(x_{toe} - x_{wj})^2 + (y_{toe} - y_{wj})^2}{(x_{toe} + x_{wj})^2 + (y_{toe} - y_{wj})^2} \right] \]  

(16)

The first derivatives are then found from these formulae:

\[
\frac{\partial \Phi_{toe}}{\partial q} = \frac{q}{k} \sum_{j=1}^{n} Q_w \left\{ \frac{\left( x_{toe} - x_{wj} \right) \left( x_{toe} + x_{wj} \right)}{\left( x_{toe} + x_{wj} \right) + \left( y_{toe} - y_{wj} \right)} + \frac{\left( y_{toe} - y_{wj} \right)}{\left( x_{toe} - x_{wj} \right) + \left( y_{toe} - y_{wj} \right)} \right\} \]  

(17)
\[
\frac{\partial^4 x_{sw}}{\partial Q_i \partial Q_j} = -\frac{1}{4\pi k} + \ln \left( \frac{(x_{sw} - x_i)^2 + (y_{sw} - y_i)^2}{(x_{sw} + x_i)^2 + (y_{sw} - y_i)^2} \right)
\]

\[
\times \left( \frac{(x_{sw} - x_i)(x_{sw} + x_i)^2 + (y_{sw} - y_i)^2}{(x_{sw} + x_i)^2 + (y_{sw} - y_i)^2} \right)
\]

\[
\times \left( \frac{(x_{sw} - x_i)(x_{sw} + x_i)^2 + (y_{sw} - y_i)^2}{(x_{sw} + x_i)^2 + (y_{sw} - y_i)^2} \right)
\]

(18)

Second derivatives also be obtained using these last equations.

6. APPLICATION.

The proposed optimization model, is applied to a specific hypothetical unconfined coastal aquifer [Liu, Cheng, Liggett, and Lee, 1981], [Taigbenu, Liggett, and Cheng, 1984]. The uncertainties in parameter estimates, boundary conditions, and imposed stresses are incorporated. The management model solutions is useful for establishing the potential applicability of the proposed model.

We examine an unconfined aquifer with \(K=70\) m/day, \(q=1\) m³/day/m, \(d=20\) m, \(\rho_s=1.025\) g/cm³, and \(\rho_f = 1\) g/cm³. Figure 1 gives an aerial view of the coast and the locations of 8 pumping wells. The well coordinates are shown in column (2) and (3) in Table 1. For each well we assume that there exist a lower bound \(Q_{low}\) and an upper bound \(Q_{up}\) for pumping rate. Equipment and operational conditions limit the upper bound. A constant \(Q_{up}=1500\) m³/day is initially chosen for all wells. We then check the critical pumping rates of each well assuming that the well exists alone. Since there is no reason that a well can pump more than the critical rate \(Q_c\), the smaller of \(Q_c\) and 1500 m³/day is used as the upper bound, which is listed in column (4) of Table 1. From economic considerations, there exists a minimum pumping rate below which the well is no longer cost-effective to operate. The value of \(Q_{low}=150\) m³/day is used for all wells as shown in column (5). With a uniform benefit rate of 0.16 $ per m³ of water and a pumping cost of 0.0024 $ per m³ per m lift of water [Gupta et al., 1996], the total optimum water withdrawals are given in Figure 1. Due to the number of wells involved and their proximity to each other, the optimization space contains a large number of local optima. In illustrative example the locations in table 1 identified with numbers 1,8 represent the possible pumping locations:

<table>
<thead>
<tr>
<th>Well Id</th>
<th>(x_w) (m)</th>
<th>(y_w) (m)</th>
<th>(Q_{up}) (m³/day)</th>
<th>(Q_{low}) (m³/day)</th>
<th>(L_i) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1100</td>
<td>1500</td>
<td>1500</td>
<td>150</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>1400</td>
<td>-1000</td>
<td>1500</td>
<td>150</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>1500</td>
<td>2400</td>
<td>1500</td>
<td>150</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>1100</td>
<td>-2000</td>
<td>1500</td>
<td>150</td>
<td>11</td>
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</tr>
<tr>
<td>7</td>
<td>1800</td>
<td>-400</td>
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<tr>
<td>8</td>
<td>1700</td>
<td>1000</td>
<td>1500</td>
<td>150</td>
<td>17</td>
</tr>
</tbody>
</table>
Table 1: Pumping wells input data for the example

Before implementing the SGA, it is necessary to use a prescribed precision for which the design variables are to be evaluated, as this determines the length of binary code required for each pumping rate. The SGA is then applied using the following parameters: population size = 20, maximum generations = 400. Different values of crossover and mutation probabilities were used and only those obtained for 0.1 for mutation probability and 0.7 for crossover probability are presented. The toe location constraint is handled using the penalty method as described previously. The optimal solution of deterministic management model given by SGA is shown in figure 1.

![Figure 1](image)

Figure 1: Saltwater intrusion front for the spatially distributed pumping wells from the unconfined aquifer and the correspondent optimal solution for their relative net benefit.

For the chance constraint management model, the freshwater outflow and the pumping rates are random variables. We assume that the coefficients of variation, defined as the normalized standard deviation $c_q = c_Q = 1\%, 5\%, 10\%$, we further assume that the pumping rate and freshwater outflow rate are not correlated $\sigma_{qQ}=0$. Results are presented to analyze the effect on pumping rates, and toe location using different reliability values. The optimal pumping pattern decreases with increasing values of $r_{toe}$. The values used for $r_{toe}$ are 90%, 95%, 99%. The results are shown in figure 2,3.
Figure 2: Saltwater intrusion front for the spatially distributed pumping wells from the unconfined aquifer and the correspondent optimal solution for their relative net benefit. The mean and standard deviation of toe location for the case $c_q = 10\%, \ c_Q= 10\%$ (4a) and the case $c_q = 1\%, \ c_Q= 10\%$ (4b) with reliability level = 90%.
Figure 3: Saltwater intrusion front for the spatially distributed pumping wells from the unconfined aquifer and the correspondent optimal solution for their relative net benefit. The mean and standard deviation of toe location for the case $c_q = 10\%$, $c_0 = 10\%$ (5a) and the case $c_q = 1\%$, $c_0 = 10\%$ (5b) with reliability level = 99\%.

The results show that the total pumping rate and his relative net benefit decreases in a nonlinear manner with increasing reliability. This implies that a larger volume of water needs to be pumped for use if we consider less risk in the intrusion distance. There are two
important observations to be noticed. The first is that if we set reliability to 99% and small variances of the coefficient of variation of the freshwater outflow, the result is the same for the deterministic optimization model. The second is that the standard deviation of toe location decreases drastically when operating at very high reliabilities for a small improvement in reliability and small variances for the coefficient of variation of the freshwater outflow.

Hence, what is considered a reasonable reliability is a decision that the engineer or manager has to make depending on the saltwater intrusion control objectives.

**CONCLUSION**

A management model with economic objective function for sustainable use of coastal aquifers under conditions of uncertainty is developed by exploiting the chance constraints programming (CCP) method. Because the values of the parameters are uncertain, the CCP method was used to enhance the accuracy of the saltwater-intruded coastal aquifers management. For the multiple pumping well problem, the analytical solution, e.g. Strack Pumping Well Solution, is most useful for engineers to conduct feasibility study and preliminary design. The optimization technique has been applied to the saltwater problem in such a way to locate either the interface or the toe position of the saltwater that entered into aquifer. The coupling of the analytical solution sharp interface approach with CCP-SGA is a very promising tool for use in solving this economic objective optimization problem. One great advantage of SGA over conventional optimization lies in its handling complex and highly nonlinear problems. With CCP, we transformed a stochastic model into its deterministic equivalent. Incorporating uncertainty into CCP-SGA model enabled us to solve the saltwater intruded coastal aquifer management problem. Using Chance constraints with genetic algorithm to solve a stochastic problem in a deterministic framework is practical tool that can be used by modelers optimizing the saltwater intruded coastal aquifers by pumping. These preliminary results demonstrate the feasibility of the approach developed coupling closeform saltwater intrusion solution, GAs, CCP...... However a better numerical solver based on real partial differential governing equation is highly needed to account for other parameters.

**REFERENCES.**


