A numerical methodology for estimating flow, saline intrusion and contamination in coastal aquifers

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ABSTRACT

We apply some advanced numerical methods to model and simulate flow, saline intrusion and contamination in the \textit{Rharb} coastal aquifer (northwestern Morocco). The numerical schemes used for saline intrusion are those developed by Esselaoui et al. [1998] for the approximation of the vertically integrated model. As for contamination, velocities are computed to be adequately incorporated into an upwind finite volume element scheme approximating the solute transport equation. The lacking field's transmissivities are restituted from the inverse problem by the new conservative identification method introduced in [EL Mansouri et al., 2000]. The considered methods have been validated through synthetic examples and comparisons. Thus the data stemmed from their application to real aquifers are reliable, at least for groundwater resources management purposes.

INTRODUCTION

The groundwater constitutes over 68\% of the world's freshwater supply. Thus it remains a vital resource for both agricultural and industrial activities in many countries. However, the development of these activities resulted in an increasing threat of groundwater by organic, inorganic and radioactive contamination. On the other hand, the droughts together with the excessive exploitation of coastal aquifers promote their pollution by saline intrusion.

Remediation and management methods bring up a grand challenge since they are prohibitively expensive and so need high accuracy predictions before proceeding their application on real sites. Hence the recourse to the improvement of the numerical modeling of flow and transport in porous media, as well as the techniques of parameters' identification, which are all essential for the simulation of groundwater pollution risks and decontamination processes.

In this paper, we try to experiment some new numerical methods on the \textit{Rharb} coastal site aquifer. The site is situated at northwestern Morocco (see Fig. 1) and represents the phreatic part of the large layered hydrological site of \textit{Rharb}. It lies as a band delimited by \textit{Sebou} river in the south and southeast, and by the \textit{bleu} lake in the north, with a coastal western edge of about 76 kilometers. Thus, the aquifer is threatened both by saline intrusion from the West Atlantic coast, and by fertilizer's infiltration inducing the transport of different pollutants as the nitrates and pesticides.

Using the conservative identification method [EL Mansouri et al., 2000], we evaluate in the following section the nodal field transmissivities that reproduce approximately the reference head repartition. Then, we compute the steady state freshwater/saltwater interface and sketch a scenario of saline intrusion using the decoupling algorithm elaborated in [Esselaoui et al., 1998]. And finally, we depict
the calculation of velocities to be inserted into an upwind finite volume element scheme extending that given for the convection diffusion equation in [Michev, 1996]. The upwinding choice being also guided by the physical fit sought in the coupling [Banton et al., 1995] with a vertical unsaturated infiltration model.

In the following, $\Omega$ stands for the horizontal cross-section of the aquifer and $T_h$ a regular triangulation of $\Omega$.

![Figure 1: Location of the Rharb coastal aquifer.](image)

CONSERVATIVE IDENTIFICATION OF FIELD TRANSMISSIVITIES

The lack of some field characteristics appearing as coefficients in the mathematical modeling by partial differential equations has always called on the investigation of inverse problems in the context of parameters' identification [Chavent, 1971]. For the transmissivity, it can be evaluated from pumping tests, but this becomes very costly in large aquifers. Giudici et al. [1995] described in the introduction of their paper most of previous works on the identification of distributed transmissivities. They also proposed a procedure restituting internode transmissivities from the inverse problem corresponding to steady or quasi-steady state flow through a two-dimensional isotropic aquifer. However, their method works only on finite differences lattices and requires two different steady state situations. Attempting to circumvent the latter limitations, we introduced in [EL Mansouri et al., 2000] the Conservative Identification (CI) method that approximates the nodal transmissivities on a triangular working mesh, using only one steady state situation.

**Conservative identification method**

In the framework of Darcy's law, we consider the steady state flow problem through an isotropic porous medium [Bear, 1972]

$$-\text{div}(T \text{ grad } h) = f, \quad \text{in } \Omega$$  

(1)
where $T(x, y)$ stands for the transmissivity coefficient, $h(x, y)$ the piezometric head and $f(x, y)$ the source term. Then, the inverse problem is formulated as follows: Given a piezometry $h(x, y)$ and its corresponding source term $f(x, y)$, find the distributed transmissivity $T(x, y)$.

Let $\Omega$ be divided into a set of control volumes $V_i$ surrounding the vertices $i$ of $T$, and made from elements' circumcenters, as sketched in Fig. 2. Then the flow through the face $\gamma_{ij} = \partial V_i \cap \partial V_j$ bisecting $e_{ij}=\{i, j\}$ is usually evaluated as follows

$$-\int_{\gamma_{ij}} T \nabla h \cdot \nu_{ij} \, ds \approx \text{mes}(\gamma_{ij}) T_{ij} \frac{(h_i - h_j)}{\text{mes}(e_{ij})}$$

where $\nu_{ij}$ is the unit normal vector on $\gamma_{ij}$ outward to $\partial V_i$ and $T_{ij}$ is the harmonic mean of $T_i$ and $T_j$, given by $T_{ij} = 2T_i T_j / (T_i + T_j)$.

The isotropy permits to switch over to the calculation of the flow through $e_{ij}$ using the same transmissivity $T_{ij}$. Consequently, the integration of (1) over the element $K = \Delta(ijk)$ displayed in Fig. 2, yields by application of Green's theorem

$$-\int_{e_{ij}} \nabla h \cdot n_{ij} \, ds - \int_{e_{jk}} \nabla h \cdot n_{jk} \, ds - \int_{e_{ki}} \nabla h \cdot n_{ki} \, ds = \int_{K} f \, d\Omega$$

where $n_{ij}$ is the unit normal vector on $e_{ij}$ outward to $K$.

So if we suppose the transmissivity given at two vertices of $K$, say $i$ and $j$, (2) reduces to a second order equation of unknown the transmissivity of the third vertex $k$. This local procedure of solving the inverse problem can be repeated by taking the output values as data for neighboring triangles.

A performing sweeping algorithm was developed to allow the computation on the whole mesh node points, after an automatic production of additional required data from the connectivity. When more pairs of experimental transmissivities are available, each subregion surrounding a pair is swept separately to minimize the error propagation.

Application to the Rharb coastal aquifer
From ten available values of experimental transmissivity, we have fixed six pairs on different field's areas by doubling some values, assuming the medium locally homogenous. The domain is partitioned into 649 triangular elements with 380 nodes, and the CI method is used through a running program which permits the computation of the transmissivity model in less than five minutes on a PC with an i486 processor. Then, new piezometric head plots are reproduced via a finite volume element scheme [Cai, 1991], to check the fitness to the reference head repartition, and estimate the reliability of restituted transmissivities. The maximal head difference is only 65.6 cm and 340 nodes have a difference less than 20 cm. Fig. 3 shows the natural amelioration of fitting between the reference and calculated heads when we use successively one pair (a), two pairs (b), three pairs (c) and six pairs(d) to determine the transmissivity. Moreover, the deduced permeabilities are in good agreement with the geological facies along the aquifer band.
Figure 3 : Improvement of fitting with the increase of experimental pairs.

SIMULATION OF SALINE INTRUSION IN THE AQUIFER

Starting from the macroscopic scale and using the conservation of mass, Darcy's law [1856] together with Dupuit's assumption and the abrupt interface approximation, allow the derivation of the vertically integrated model [Bear, 1979] that couples the freshwater and saltwater equations through a dynamic relation at the interface. For an unconfined isotropic aquifer, the problem is written as follows
\[
\begin{cases}
S^f \frac{\partial h^f}{\partial t} - \theta \frac{\partial h^i}{\partial t} - \nabla \cdot (T^f \nabla h^f) = r^f, \quad \text{in } \Omega_i \times [0, T] \\
\theta \frac{\partial h^i}{\partial t} - \nabla \cdot (T^i \nabla h^i) = 0, \quad \text{in } \Omega_i \times [0, T] \\
h^i = \alpha_i h^s - \alpha_f h^f, \quad \text{in } \Omega_i \times [0, T] 
\end{cases}
\]

where

$\Omega_i$ : domain part generating the interface presence.
$S^f$ : effective porosity.
$\theta$ : porosity.
$T^f = (h^f - h^i) K$ : freshwater transmissivity, $[L^2 T^{-1}]$.
$T^s = (h^s - \sigma) K$ : saltwater transmissivity, $[L^2 T^{-1}]$.
$K$ : permeability, $[LT^{-1}]$.
$\sigma$ : substratum elevation, $[L]$.
$h^f$ : freshwater piezometric head, $[L]$.
$h^i$ : saltwater piezometric head, $[L]$.
$h^s$ : interface elevation, $[L]$.
$r^f$ : freshwater surfacique refill, $[LT^{-1}]$.
$\alpha_s = \rho_s/(\rho_s - \rho_f)$, $\alpha_f = \rho_f/(\rho_s - \rho_f)$
$\rho_s$ and $\rho_f$ being the specific weighs of saltwater and freshwater respectively.

Polo and Ramis [1983] proposed for the resolution a finite differences based numerical method using the indirect toe tracking introduced by Wilson and Costa [1982]. In a recent work [Esselaouï et al., 1998], an advanced investigation of the coupled and decoupled resolutions by finite elements is presented with various tests and comparisons.

**Steady state interface**

By the Ghiben-Herbég approximation [Bear, 1972], the steady state is assumed to be reached when the saltwater piezometric head is equal to the sea level everywhere on $\Omega_i$. In this case, the problem reduces to the freshwater flow equation and the interface boundary condition is merely

$h^i = -\alpha_f h^f, \quad \text{in } \Omega_i \times [0, T]$

Thus, concerning the aquifer at hand, the interface is computed explicitly from the available head. As for saltwater toe, it is detected along the nearest points to the coast satisfying: $h^i = -\sigma/\alpha_f$.

Fig. 4 presents the contour plots of the steady state interface for the Rharb coastal aquifer in 1992. It is observed that the maximum seawater intrusion is located in three regions (south, middle and north) of the aquifer, where the interface elevation exceeds -20 m.
Figure 4: Steady state interface in 1992.

Figure 5: Simulation of interface motion.
Saline intrusion

To simulate the unsteady interface, we first derived the nodal permeabilities from the identified freshwater transmissivities and the steady state interface by

\[ K = \begin{cases} T'/l(h' - h) & \text{if the node is seaward of the toe.} \\ T'/l(h' - \sigma) & \text{if the node is landward of the toe.} \end{cases} \] (4)

The porosity and effective porosity are chosen as mean values on the whole domain and are respectively \( \theta = 0.2 \) and \( S' = 0.05 \).

Considering the steady interface as initial state, and assuming a continuous drought, and doubling the rates in all pumping wells, we have run the decoupling algorithm described in [Esselaoui et al., 1998] with \( \Delta t = 10 \) days. Once again, the scheme provides a good convergence in less than five iterations each time step. Fig. 5 shows the simulation of freshwater/saltwater interface in 1996 and in 2000. Indeed, the decrease of refills entails a landward movement of the interface.

CONTAMINATION BY BRINE TRANSPORT

A two-dimensional transient solute transport process in saturated porous medium is described by the equation [Bear, 1979]

\[ \frac{\partial (\theta b c)}{\partial t} - \text{div} (\theta b D(U) \nabla c - \theta b U c) = Q c' \] (5)

where

- \( b \) : freshwater saturated thickness, [L].
- \( c \) : concentration of solute, [ML^{-3}].
- \( U = (u,v)^T \) : average velocity vector of fluid in porous medium, [LT^{-1}].
- \( D(U) \) : hydrodynamic dispersion coefficient tensor, [L^2T^{-1}].
- \( Q \) : strength of a source or sink well, [LT^{-1}].
- \( c' \) : concentration in the pumped or injected fluids [ML^{-3}], such that \( c = c' \) for a pumping well, and \( c' \) is specified at an injection well.

For an isotropic porous medium, the four components of tensor \( D(U) \) may be expressed by [Bear, 1979]

\[ D_{xx} = \alpha_L \frac{u^2}{|U|} + \alpha_T \frac{v^2}{|U|} + D^* \quad , \quad D_{yy} = \alpha_T \frac{u^2}{|U|} + \alpha_L \frac{v^2}{|U|} + D^* \]

\[ D_{xy} = D_{yx} = (\alpha_L - \alpha_T) \frac{uv}{|U|} \]

where \( \alpha_L \) and \( \alpha_T \) are the longitudinal and transversal dispersivities [L], respectively, and \( D^* \) is the coefficient of molecular diffusion [L^2T^{-1}], and are all characteristics of the porous medium. We assume here that \( \alpha_L = 40 \, m \), \( \alpha_T = 5 \, m \) and \( D^* = 10^{-5} \, m^2 / d \).

The spatial discretisation method used here follows the upwind finite volume element scheme detailed in [Michev, 1996] for the convection diffusion equation. So, considering the notations related to the dual finite volume mesh handled above, the
integration of (5) over a control volume $V_i$ with the application of Green's theorem provides
\[
\frac{mes(V_i) b_i}{\Delta t} \sum_{j \in \omega_i} \frac{b_j}{2} - mes(\gamma_{ij}) \left\{ D(U_{ij}) \nabla c_{ij} - \frac{U_{ij} v_{ij} + \left| U_{ij} v_{ij} \right|}{2} c_i - \frac{U_{ij} v_{ij} - \left| U_{ij} v_{ij} \right|}{2} c_j \right\} = \frac{mes(V_i) Q_i c'_i}{\theta}
\]
where
\[
\omega_i = \{ j : i \text{ and } j \text{ are distinct vertices of the same triangle} \}.
\]
\(\Delta t\) : time step.
\(\bar{c}\) : concentration of a previous iteration.
\(c_i, \bar{c}_i, c'_i, b_i, \text{ and } Q_i\) : values at node \(i\).
\(U_{ij} = K_{ij} (\nabla h)_{ij} / \theta\).
\(K_{ij} = 2 K_i K_j / (K_i + K_j)\) : harmonic mean of the nodal permeabilities given by (4).
\(\nabla h)_{ij} = \frac{1}{2} \left( \text{card}(t_i)^{-1} \sum_{K \in t_i} \nabla h_{iK} + \text{card}(t_j)^{-1} \sum_{K \in t_j} \nabla h_{jK} \right)\)
\(t_i = \{K \in T_h : i \text{ is a vertex of } K\}\).

In a first application, we neglect the solute transferred to the aquifer system by infiltration, so that only the concentrations of brine observed along the eastern edge contribute to the contamination. This can be viewed as a minor quantification of solute in the aquifer. The major pollution is estimated by taking the value of 4 g/l as a constant Dirichlet boundary condition along the thick edge schematized in Fig. 6-a. The contaminant plume spread is simulated until the year 2010. Fig. 6-b indicates the augmentation of brine concentrations in four points (B, C, D and E) of the aquifer situated at approximately two kilometers off the source of pollution. The high spread observed at points D and E is due to the important convection induced by the flow direction. As for B and C, the spread is relatively slow, for it is practically caused by the dispersion.

Figure 6 : Simulation of brine transport.
CONCLUSION

We displayed the importance of some mathematical numerical contributions for the groundwater resources development. Indeed, a reliable transmissivity model was determined thanks to an efficient conservative identification method, which was carried into effect in order to avoid the hard manual adjustment procedure. Moreover, the storage memory is actually optimized in decoupling the resolution of the freshwater/saltwater interface. Another hint is the new conservative formula provided to compute flow velocities from an observed head repartition. These discrete velocities are shown to be essential in tracking contaminant transports by the robust general finite volume schemes. Our contributions were tested in some previous numerical works before being successfully applied here to a coastal site of Morocco. Through the presented simulations, we attempted in a sense to prompt the concerned Moroccan authorities to consider such pollution threats. But we also demonstrated the sublime usefulness of modern numerical hydrology for a cheap preservation of the groundwater resource.

References


Key words: FVE methods, transmissivity, saline intrusion, contamination

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